Impact Point Dispersion Prediction for R-Han 300 Artillery Rocket Using Monte Carlo

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Abstract

The effectiveness of artillery rocket in battlefield is determined by its impact point dispersion, which may occur due to manufacturing and measurement inaccuracy, initial launch perturbations and atmospheric conditions. Therefore, the objective of this study is to establish model that could predict the impact point dispersion of R-Han 300 rocket using Monte Carlo method. A Generic 6 Degree-of-Freedom equation of motion model was implemented to investigate the impact point. Initially two simulations with 1000 iterations were carried out. The first one to study the effect of value uncertainty of every parameter on the impact point dispersion at launch elevation angle 50 degrees. The second one to study the impact point dispersion caused by value uncertainty of all the parameters at launch elevation angles ranging from 30 to 70 degrees. The second simulation is then repeated with 10000 iterations. This study showed that the dispersion increases as the initial launch elevation angle increases, except around the optimal launch elevation angle that gives the farthest range. Monte Carlo simulation with 10000 iterations showed a better normal distributed data than the simulation with 1000 iterations. The maximum difference in value of circular error probable (CEP) resulted from both simulations is very small, which is 3.16%.

Keywords: impact point dispersion, Monte Carlo simulation, R-Han 300.

1. Introduction

R-Han 300 is a 300 mm caliber artillery rocket design which is currently an object of research at UNHAN (Muslimin, Triharjanto, and Ruyat 2022), where as an artillery rocket, one of its performance parameters is range (Dali et al. 2019). Rocket experiences disturbances during its flight, so without a guidance system, it will deviate from its nominal trajectory (Nugroho et al. 2021). Dispersion is the distribution of the impact point around the distribution center point (Song et al. 2019). It is a measure of deviation of the trajectory from its nominal value (Luo 2015). Due to its dispersion, the unguided artillery rocket is used as an area target weapon (Ozog, Jacewicz, and Glebocki 2020). The prediction of its impact point dispersion is needed to determine its effectiveness (Katsev 2018).

Impact point dispersion occurs because of the uncertainty in the values of the parameters that affect its flight trajectory (Wiputgasemsuk 2021). Uncertainty of initial condition parameter values when exiting the launcher is one that may occur (Raza and Wang 2022). Fabrication and measurement inaccuracies will lead to measurement uncertainty for the mass and inertia (Le and Konecny 2021). Atmospheric conditions that are always changing cause uncertainty in measurement of atmospheric parameter (Trzun and Vrdoljak 2020). Production process of solid propellant is a source for variations of thrust curve (Fernandes, Sauto, and Pirk 2020).

Monte Carlo simulation has been used in various analysis of rocket flight performance. Monte Carlo was used to predict impact point for guided artillery rocket (Ozog et al. 2020), (Raza and Wang 2022), and (Wiputgasemsuk 2021). The dispersion of unguided and guided rocket at launch angles from 30 to 50 degrees was studied (Glebocki and Jacewicz 2020). The effect of inaccuracy and fabrication quality on the dispersion of rocket trajectory was...
investigated (Trzun, Vrdoljak, and Cajner 2021). The effect of thrust misalignment and mass inaccuracy on the unguided rocket impact point was studied (Le and Konecny 2021). The contribution of this paper is that the analysis was carried out with more variations for the launch elevation angle ranging from 30 to 70 degrees, each was carried out with 1000 and 10000 times iterations for a case study of an unguided artillery rocket which optimal range is 92.7 km.

The purpose of this study is to investigate the dispersion of impact point of artillery rockets resulted from the value uncertainty of several parameters that affect its flight trajectory at launch elevation angle ranging from 30 to 70 degrees. The dispersion of impact point is required as a reference for the area of impact of the rocket. This analysis was carried out using a Monte Carlo simulation combined with six degrees of freedom (6-DOF) rocket flight trajectory simulation. In this study, Monte Carlo simulation was carried out in two cases, namely with 1000 and 10000 iterations. The results of both cases then compared.

2. Methodology

2.1. Rocket Configuration

The rocket model in this study is the R-Han 300, which is a rocket designed to replace the SS-80 Astros rocket used by the TNI. The rocket adopted the materials and technology used in the R-Han 122 B rocket developed by the Ministry of Defense (Kemhan), and the RX-320 developed by the Research Organization for Aeronautics and Space of the National Research and Innovation Agency (ORPA BRIN) (Riyadi 2022).

The geometry and dimensions of the R-Han 300 rocket are shown in Figure 2-1. The total length of the rocket is 5.6 m, the diameter is 0.306 m, the length of the nose cone is 1.071 cm, the width of the fin is 0.24 m and the root chord of the fin is 0.30 m. The R-han 300 properties value such as mass, moment of inertia and thrust are given at Table 2-1 (Riyadi 2022).

![Figure 2-1: Geometry and dimensions R-Han 300](image)

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Initial mass</td>
<td>707 kg</td>
</tr>
<tr>
<td></td>
<td>Final mass</td>
<td>381 kg</td>
</tr>
<tr>
<td>2.</td>
<td>Propellant Mass</td>
<td>326 kg</td>
</tr>
<tr>
<td>3.</td>
<td>Initial moment of inertia at xb, yb, zb axis</td>
<td>21 kg.m², 1827 kg.m², 1827 kg.m²</td>
</tr>
<tr>
<td>4.</td>
<td>Final moment of inertia at xb, yb, zb axis</td>
<td>16 kg.m², 1413 kg.m², 1413 kg.m²</td>
</tr>
<tr>
<td>5.</td>
<td>Average thrust</td>
<td>60000 N</td>
</tr>
<tr>
<td>6.</td>
<td>Burning time</td>
<td>11.45 seconds</td>
</tr>
</tbody>
</table>

2.2. Trajectory Simulation

The rocket trajectory simulation was made using Matlab Simulink. The schematic of the trajectory simulation model is shown in Figure 2-2. This model cover the mathematical models of rocket, atmosphere, aerodynamic, propulsion, mass, and moment of inertia.
In this simulation, the rocket is assumed to be a rigid body, axisymmetric, with mass and moment of inertia that decreases linearly with time during powered flight. The simulation is made for a three-dimensional space with six degrees of freedom (6-DOF). This 6-DOF simulation is used to calculate the effect of value uncertainty of various parameters on the rocket's impact point.

The coordinate system used in this model is shown in Figure 2-3 (Glebocki and Jacewicz 2020). U, V and W and P, Q, R are rocket translation velocities and angular velocity of the rocket in body xb, yb and zb axis respectively, and m is rocket mass. X, Y, Z are the position coordinates of the rocket center of mass, and , , are Euler angles (roll, pitch and yaw).

Mathematical model for a 6-DOF rocket motion is very common, and published in many publications. The mathematical model used in this study referred to previous work (Szklarski, Glebocki, and Jacewicz 2020) and (Glebocki and Jacewicz 2020). The aerodynamic force coefficient was calculated as a function of the angle of attack and Mach number. Atmospheric parameters such as air density, pressure and temperature are calculated using International Standard Atmosphere (ISA) model which can be found in previous publication (Osaci 2018). The mathematical equations used in the rocket trajectory simulation are as follow:

\[
\begin{bmatrix}
\dot{U} \\
\dot{V} \\
\dot{W}
\end{bmatrix}
= \frac{F}{m}
\begin{bmatrix}
0 & -R & Q \\
R & 0 & -P \\
-Q & P & 0
\end{bmatrix}
\begin{bmatrix}
U \\
V \\
W
\end{bmatrix}
\]  

(1)
Equations 1-4 is the 6-DoF equations of motion of the rocket. These formulas determine the trajectory of a rocket. Equations 5-6 determine the moment inertia, forces and moments acting on the rocket. Ix, Iy, Iz are moments of inertia. Mx, My, Mz are aerodynamic moments acting on the rocket body. Ax, Ay, Az are aerodynamic forces, gx, gy, gz are gravitational forces, and Th is propulsive thrust. The mass and inertia of the rocket will decrease during the propellant combustion process. The mass and inertia of the rocket are expressed as follows.

\[
m = m_0 - \frac{m_p}{t_b}t \quad , \quad t \leq t_b \tag{8}
\]

\[
m = m_e \quad , \quad t > t_b \tag{9}
\]

\[
I_{yy} = I_{yyo} - \left(\frac{I_{yyo} - I_{yye}}{t_b}\right)t \quad , \quad t \leq t_b \tag{10}
\]

\[
I_{yy} = I_{yye} \quad , \quad t > t_b \tag{11}
\]

The initial mass of the rocket is mo and Iyyo are the moment of inertia of the rocket with the propellant present, me and Iyye are the mass and moment of inertia of the rocket without the propellant, and tb is the burning time of the propellant. During the propellant combustion process the mass and moment of inertia of the rocket will decrease according to equations 8 and 10. After the propellant combustion is complete, the mass and moment of inertia of the rocket will be constant according to equations 9 and 11.
2.3. Monte Carlo Simulation

Monte Carlo simulation is a method that is widely used for predicting the dispersion of a rocket's impact point (Junior et al. 2022). Monte Carlo simulation is a method for determining how the simulation output is affected by the uncertainty of the input parameters (Jacewicz et al. 2022). This method is ideal for investigating the effect of a combination of various parameters on rocket's performance (Noga, Michalow, and Ptasinski 2021). Monte Carlo simulation is carried out by taking sample of each uncertain input parameter, then simulating a physical model to determine the desired output. This simulation is repeated to create an empirical probability distribution of the output variables. The more the number of iterations, the more accurate the results will be (Trzun et al. 2021).

The Monte Carlo simulation results are valid for standard artillery rockets if the output value distribution shows a normal distribution (Moon and Gordis 2021). Normal distribution is a probability distribution that showing data near the mean are more frequent in occurrence than data far from the mean.

The flow of Monte Carlo simulation is shown in Figure 2-4 (Trzun et al. 2021). As shown in Figure 4, the Monte Carlo simulation is basically a trajectory simulation but with addition of random uncertainty values of some parameters as input. The maximum limit of random uncertainty value of each parameter is described Table 2-2.

One parameter to be calculated after Monte Carlo simulations is circular error probable (CEP). CEP is calculated as a measure of dispersion and accuracy of the rocket (Zhang et al. 2017). CEP is defined as the circle consisting of 50% of the impact points and centered at the mean impact point (Wiputgasemsuk 2021). Value of CEP is estimated as a median from distribution of radius dispersion of impact points (Trzun and Vrdoljak 2021).

Monte Carlo simulation in this study was carried out in two steps. The first step was carried out to calculate and observe the effect of value uncertainty of each parameter on the impact point dispersion at launch elevation angle 50 degrees. Simulations were carried out with 1000 times iterations for each parameter. The second step was then carried out to calculate the effect of value uncertainty of all parameters on the impact point dispersion at launch elevation angle ranging from 30 to 70 degrees. Simulations were carried out with 1000 times iterations for each launch elevation angle. The second step then repeated with 10000 times iterations for each launch elevation angle. CEP values were calculated for each launch elevation angle and for both simulations of the second step.

2.3. Parameter Uncertainty Value

There are many parameters due to their uncertainty values may disturb the trajectory of a rocket, but eight parameters were selected in this study as source of disturbances. These parameters are frequently and almost unavoidable happened in real flight. These eight
parameters are initial velocity, launch elevation angle, launch azimuth angle, moment of inertia, mass, wind, and aerodynamic. The aerodynamic parameter consists of axial force coefficient \((C_a)\), normal force coefficient \((C_n)\), and pitch moment coefficient \((C_m)\). The value uncertainty of these parameters are shown in Table 2-2. The aerodynamic values uncertainty is determined based on previous studies (Nguyen et al. 2014) and (Charubhun, Chusilp, and Nutkumbang 2011), the values uncertainty of wind speed and direction are obtained from observations at the launch site, while the values uncertainty of other parameters were determined based on previous study (Mihailescu, Radulesscu, and Coman 2011).

Table 2-2: Parameter’s uncertainty value

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter</th>
<th>Nominal Value</th>
<th>Uncertainty Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Initial velocity</td>
<td>20 m/s</td>
<td>± 2 m/s</td>
</tr>
<tr>
<td>2.</td>
<td>Elevation</td>
<td>30 – 70 degrees</td>
<td>± 1 degree</td>
</tr>
<tr>
<td>3.</td>
<td>Azimuth</td>
<td>0 degrees</td>
<td>± 1 degree</td>
</tr>
<tr>
<td>4.</td>
<td>Moment of inertia</td>
<td>21, 1827 kg.m²</td>
<td>± 1 %</td>
</tr>
<tr>
<td>5.</td>
<td>Total mass</td>
<td>707 kg</td>
<td>± 1 %</td>
</tr>
<tr>
<td>6.</td>
<td>Wind</td>
<td>0 m/s</td>
<td>Speed : ± 2 m/s</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Direction : 1-360 degrees</td>
</tr>
<tr>
<td>7.</td>
<td>Thrust</td>
<td>60000 N</td>
<td>± 1 %</td>
</tr>
<tr>
<td>8.</td>
<td>Aerodynamic</td>
<td>Based on table in simulation</td>
<td>± 2% for (C_a), ± 5% for (C_n), ± 10% for (C_m)</td>
</tr>
</tbody>
</table>

3. Result and Analysis

The nominal range of R-Han 300 for various launch angles is shown in Figure 3-1. The maximum range is 92.7 km, achieved at launch elevation angle 65 degrees. Figure 3-2 shows the effect of value uncertainty of various parameters individually on impact point dispersion at launch elevation angle 50 degrees. The x-axis is range dispersion and the y-axis is the cross-range dispersion. The impact point dispersion as shown in Figure 3-2 is strongly influenced by the determination of the range of parameter’s uncertainty value. Since R-Han 300 is now in design phase, the range of parameter’s uncertainty value is determined based on works conducted by other authors. For real rocket, then the value depend on the quality of manufacture, assembly and measurement.

Figure 3-1: Ranges at several launch elevation angles.
Figure 3-2: Impact point dispersion at launch elevation angle 50 degrees generated by the value uncertainty of (a) initial velocity, (b) elevation, (c) azimuth, (d) moment of inertia, (e) total mass, (f) wind, (g) thrust and (h) aerodynamic.

Figure 3-2 shows the range and cross-range dispersion due to parameter’s uncertainty value. The largest dispersion on the x-axis is resulted by the value uncertainty of launch elevation angle, in which the difference between the maximum and minimum values of dispersion reaches 5077 meters. The value uncertainty of the thrust generates the second largest dispersion on the x-axis, in which the range of dispersion reaches 4087 meters. The smallest dispersion is resulted by the value uncertainty of the moment of inertia, in which range of dispersion is only 116 meters. On the y-axis there are only two parameters that have contribution to dispersion, namely the value uncertainty of the launch azimuth angle and the wind. The largest dispersion is generated by the value uncertainty of the wind, in which range of dispersion is 5908 meters.

The combination effect of value uncertainty of all parameters on R-Han 300 impact points dispersion at launch elevation angle ranging from 30 to 70 degrees are shown in Figure 3-3 and Table 3-1. The simulations were carried out with 1000 iterations for each launch elevation angle. As shown in Table 3-1, the magnitude of the dispersion on the x-axis and y-axis increase as the launch elevation angle increases, except on the x-axis for the launch elevation angle ranging from 55 to 65 degrees, where the magnitude of dispersion on the x-axis decreases. Table 3-1 and Figure 3-3 also showed that at lower launch elevation angle, the dispersion on the x-axis is much larger than the dispersion on the y-axis, but at launch elevation angle 60 degrees, the magnitude of the dispersion on the y-axis is became slightly larger than the dispersion on the x-axis, and became more larger at the launch elevation angle 65 and 70 degrees.
As mentioned previously, the dispersion on x-axis and y-axis increase as the launch elevation angle increases, except on the x-axis for the launch elevation angle ranging from 55 to 65 degrees, where the magnitude of dispersion on the x-axis decreases. This is because at launch elevation angles ranging from 55 to 65 degrees, the change in range due to launch elevation angle is small (as shown in Figure 3-1), so the contribution of value uncertainty of launch elevation angle on dispersion became smaller. The trend of how dispersion changes as the launch elevation changes is quite similar to the result of studies conducted by Glebocki and Jacewicz (2020) and Mihailescu, Radulesscu, and Coman (2011), but Glebocki and Jacewicz (2020) in their work only demonstrated for launch elevation angle ranging from 20 to 50 degrees for a 122 mm artillery rocket with maximum range 40 km, and Mihailescu, Radulesscu, and Coman (2011) demonstrated for launch elevation angle ranging from 30 to 50 degrees for an extinguished fire rocket with maximum range 4.8 km. The data distribution of range and cross-range dispersions are shown in Figure 3-4 and Figure 3-5.
To show that the Monte Carlo simulation results are valid for standard artillery rockets, the output value distribution must show a normal distribution (Moon and Gordis 2021) and (Trzun and Vrdoljak 2020). This can be visually investigate through the data distribution of the output value (Trzun and Vrdoljak 2020). As shown in Figure 3-4 and Figure 3-5, visually...
it is clear that the distribution of the all output values showing data near the mean are more frequent in occurrence than data far from the mean, and hence indicated a normal distributed data.

Another simulations for the combination effect of value uncertainty of all parameters on impact point dispersion were carried out, but with 10000 iterations. Table 3-2 shows the value of CEP generated from both simulations with 1000 and 10000 iterations. Table 3-2 shows that the value of CEP generated from simulation with 1000 iterations is only slightly different from the value generated from simulation with 10000 iterations. At several launch elevation angles, the differences are below 1%. The maximum difference in CEP value is 3.16% at launch elevation angle 65 degrees. Table 3-2 also shows a trend that the value of CEP increases as the launch elevation increases. This trend is similar to the result of work did by Glebocki and Jacewicz (2020).

**Table 3-2: Area of dispersion**

<table>
<thead>
<tr>
<th>Launch Elevation (degrees)</th>
<th>CEP 1000 iterations [m]</th>
<th>CEP 10000 iterations [m]</th>
<th>Delta CEP [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1005</td>
<td>977</td>
<td>-2.78%</td>
</tr>
<tr>
<td>35</td>
<td>1069</td>
<td>1076</td>
<td>0.65%</td>
</tr>
<tr>
<td>40</td>
<td>1342</td>
<td>1342</td>
<td>0.00%</td>
</tr>
<tr>
<td>45</td>
<td>1728</td>
<td>1744</td>
<td>0.93%</td>
</tr>
<tr>
<td>50</td>
<td>2149</td>
<td>2152</td>
<td>0.14%</td>
</tr>
<tr>
<td>55</td>
<td>2461</td>
<td>2399</td>
<td>-2.52%</td>
</tr>
<tr>
<td>60</td>
<td>2515</td>
<td>2517</td>
<td>0.08%</td>
</tr>
<tr>
<td>65</td>
<td>2533</td>
<td>2613</td>
<td>3.16%</td>
</tr>
<tr>
<td>70</td>
<td>2839</td>
<td>2845</td>
<td>0.21%</td>
</tr>
</tbody>
</table>

The data distribution of ranges and cross-ranges dispersions for launch elevation angles ranging from 30 to 70 degrees and for simulation with 10000 iterations are shown in Figure 3-6 and Figure 3-7. These figures show that the data distribution of ranges and cross-ranges for simulation with 10000 times iterations are more symmetric to the mean value, and most data are near the mean, which indicate a better normal distributed data.
Figure 3-6: Frequency of range for 10000 iterations.

Figure 3-7: Frequency of cross-range for 1000 iterations.

4. Conclusions

A Monte Carlo simulation model to predict the effect of disturbances on impact point dispersion of an artillery rocket has been developed for the R-Han 300. Eight parameters were selected in this study as source of disturbances, namely initial velocity, launch elevation
angle, launch azimuth angle, moment of inertia, mass, wind, and aerodynamic. The results showed that each parameter disturbance generates dispersion uniquely. The launch elevation angle have strongest influence on dispersion in range, while dispersion in cross-range was strongly influenced by the wind.

The range of dispersion increases as the launch elevation angle increases, except near the optimal launch elevation angle that gave the farthest range. At this optimal elevation and beyond the cross-range dispersion is became larger than the range dispersion. Monte Carlo simulation with 10000 iterations shows a better normal distributed data, which means more accurate than the simulation with 1000 iterations. The maximum difference in value of CEP generated from both simulations is 3.16% at launch elevation angle 65 degrees.

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Contributorship Statement

AR is contributed for developing the simulations and preparing the manuscript. RHT and PW are contributed for analysis, review and editing the manuscript.

References


