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Automatic Gain Control for UAV Stability Augmented System Using Jacobian Method

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Abstract

A flight control system is an important system in aircraft to direct the aircraft and maintain the aircraft's attitudes. One of the most important parts of the flight system is the stability augmentation system designed to improve the stability of the aircraft during flight. This system is usually located as the most inner-loop part of the flight control system. There are many methods to design a gain control feedback for aircraft stability systems. The most common method is the classical PID tuning which contains trials and error. To verify the gain, an analysis of the closed loop system must be conducted to ensure that the aircraft response is within the design requirements. This may happen several times in a single trim point before correct gain feedback is obtained. In this paper, the longitudinal stability augmentation system for Unmanned Aerial Vehicle Albatross is designed using the Jacobian methods. The Jacobian method is capable to generate feedback value automatically within the limitation given by design requirements and objectives. Therefore, this method allows a faster design for the stability augmented system especially on multi-trim points models. In this paper, the stability augmentation system is designed in a trimmed cruise condition. The results show that the new system gives damping for short period ζ_{sp} = 0.85, improving the original damping ζ_{sp} = 0.80. The simulation results for elevator step input also show that the designed system gives a rise time of 0.023 s and a settling time of 33.042 s compared to the classical PID which gives rise a time of 0.052 s and a settling time of 32.245 s.

Keywords: *UAV; flight control; longitudinal; Jacobian; Stability.*

Nomenclature

d	=	Distance between thrust line and c.g., m
g	=	Gravity acceleration, m/s^2
m	=	Mass, kg
D	=	Drag force, N
L	=	Lift force, N
M_A	=	Aerodynamic moment, Nm
S	=	Wing reference area, M^2
T	=	Thrust, N
V	=	Freestream velocity
$\boldsymbol{A}$	=	State matrix
$\boldsymbol{B}$	=	Input matrix
$\boldsymbol{C}$	=	Output matrix
$\boldsymbol{D}$	=	Feedforward matrix

$\boldsymbol{I}$	=	Identity matrix
$\bar{u}$	=	Input vector
$\bar{x}$	=	State vector
$\bar{y}$	=	Output vector
α	=	Angle of attack, rad
γ	=	Flight path angle, rad
δe	=	Elevator deflection, rad
λ	=	Eigenvalues
ρ	=	Freestream density, kg/m^3
τ	=	Thrust line inclination angle, rad

1. Introduction

A flight control system is an essential system in an aircraft to direct the aircraft and maintain attitudes. A conventional flight control system for fixed-wing aircraft consists of flight control surfaces, cockpit controls, connecting linkages, and the necessary operating mechanisms to control aircraft direction during a flight. In an Unmanned Aerial Vehicle (UAV), the pilot command cannot be issued from the aircraft cockpit, but instead from the ground control. In this case, the pilot's view and the aircraft's view will be different and the pilot will have a harder time sensing the aircraft's direction and attitude. Since the cockpit also moved to the ground there is also a delay issue for the aircraft sensor system to be transmitted to the ground. Therefore the existence of the flight control system is very crucial to ensure the UAV's level of stability.

To design a flight control, generally, a mechanical model based on the actual aircraft is needed. The quality of the flight control depends heavily on the accuracy of the model. Some research has been conducted on the modeling process of fixed-wing UAVs (No et al., 2005; Öner et al., 2012). There are several methods to design flight control systems. Among them, the most known flight control method for fixed-wing UAVs is the traditional PID control. This method is popular because it has a simple control principle, easy-to-modify parameters, and low requirements for model accuracy (Bao et al., 2021). Linear quadratic regulator (LQR) can be applied to system dynamics where the cost is described by quadratic function (Harno, 2019; Rinaldi et al., 2013; Vinokursky et al., 2019). Adaptive control has been used in a collision avoidance system (Lu et al., 2022) and formation flight (Schumacher & Rajeeva Kumar, 2000; Yu et al., 2018). Other methods include neural control (Bu et al., 2015), linear tracking control (Yeonsik Kang & Hedrick, 2009), dynamic inversion control (Wang et al., 2013), etc. In this paper, the longitudinal stability augmentation system for Unmanned Aerial Vehicle Albatross is designed using the Jacobian methods. The Jacobian method is capable to generate feedback value automatically within the limitation given by design requirements and objectives. Therefore, this method allows a faster design for the stability augmented system especially on multi-trim points models.

Albatross is a fixed wing-UAV with an inverted V-tail. This UAV is used for research purposes in the aeronautics technology center – BRIN. The Albatross can fly for up to 4 hours, boasts a 10 kg MTOW, and is entirely open and customizable, making it as ideal for boundary-pushing research and development projects as it is for commercial operations. To date the Albatross can be found in many countries, performing a range of applications from marine life conservation and humanitarian aid to asset management, surveillance, and precision agriculture. Albatross has a sleek and durable composite frame comprised of carbon fiber and fiberglass. It has two electrically powered propeller engines attached to the wing. Albatross has fixed tricycle-type landing gear. The albatross figure is shown in Figure 1-1 and the detailed specification of the UAV can be seen in Table 1.

Figure 1-1: Albatross UAV in Aeronautics Technology Center-BRIN.

Table 1-1: UAV Albatross Specifications

Parameters	Value
Wingspan	3 m
Endurance	Up to 4 hrs
Range	Up to 280 km
Cruise Speed	18 m/s
Max Level Speed	40 m/s
MTOW	10+ kg
Material	Carbon fiber and fiberglass composite

This paper presents the design of a longitudinal stability augmentation system for UAV Albatross using Jacobian methods. The Jacobian method is capable to generate feedback value automatically within the specified limits. The limitation might be get from the design requirements and objectives (DRO) or regulations. This method allows a faster design for the stability augmentation system on multi-trim points compared to the classical method such as PID tuning. The flight control is designed on several trim points. The trim points are obtained from the linearized model of the aircraft. In this paper, the pilot dynamic is not considered.

2. Basic Theory

2.1. State Space

State space is a mathematical model of a physical system as a set of input, output, and state variables related by first-order differential equations. State variables are variables whose values evolve over time in a way that depends on the values they have at any given time and on the externally imposed values of input variables. Output variables values depend on the values of the state variables. When an aircraft is in a trimmed condition, it can be modeled as a linear state space system in the following form:

$$\dot{\bar{\boldsymbol{x}}}(t) = \boldsymbol{A}\,\bar{\boldsymbol{x}}(t) + \boldsymbol{B}\,\bar{\boldsymbol{u}}(t) \tag{2-1}$$

$$\bar{\boldsymbol{y}}(t) = \boldsymbol{C}\,\bar{\boldsymbol{x}}(t) + \boldsymbol{D}\,\bar{\boldsymbol{u}}(t) \tag{2-2}$$

The system characteristic can be observed from the eigenvalues of the matrix $\boldsymbol{A}$ or by constructing the characteristic polynomial found by taking the determinant of $s\boldsymbol{I} - \boldsymbol{A}$.

$$\lambda(s) = |s\boldsymbol{I} - \boldsymbol{A}|) \tag{2-3}$$

The roots of this polynomial are called poles and equivalent to eigenvalues of matrix $\boldsymbol{A}$. These poles can be used to analyze the stability and natural response of the dynamic system.

2.2. Aircraft Longitudinal Dynamic Stability Modes

Generally, the longitudinal motion includes every aircraft motion which can be observed from the side views. In this mode, there are two kinds of oscillatory motion experienced by the aircraft which are short periods and phugoids. Both longitudinal dynamic stability modes are excited whenever the aircraft is disturbed from its equilibrium trim state. A disturbance may be initiated by pilot control inputs, a change in power setting, airframe configuration changes such as flap deployment, and external atmospheric influences such as gusts and turbulences.

Short-period mode is a damped oscillation in the pitching motion. The short-period mode generally has a short period (hence the name). This mode can be excited whenever the aircraft is disturbed from its pitch equilibrium state. This mode can be described as a second-order oscillation in which the principal variables are the angle of attack α, pitch rate q, and pitch attitude θ. A unique trait of this mode is that the speed remains approximately constant (u = 0) during a disturbance. The stability limits in this mode are quantified in terms of maximum (ζmax) and minimum damping ratio (ζmin).

Phugoid is a longitudinal oscillatory motion that has a longer period. This mode generally has a large amplitude with variations of airspeed, pitch angle, and altitude, but almost no change in the angle of attack. The phugoid mode appearance is like a sinusoidal flight path about the trimmed height datum.

2.3. Root Locus

The root locus is a graphical representation of a feedback system in the complex s-plane. The root locus shows the possible locations of closed-loop poles by varying values of a certain system parameter. In addition to determining the stability of the system, the root locus can be used to design the damping ratio (ζ) and natural frequency (ωn) of a feedback system. Lines of constant natural frequency can be drawn radially from the origin and lines of constant natural frequency can be drawn as arccosine whose center points coincide with the origin. By selecting a point along the root locus that coincides with the desired damping ratio and/or natural frequency, a gain K can be calculated and implemented in the controller.

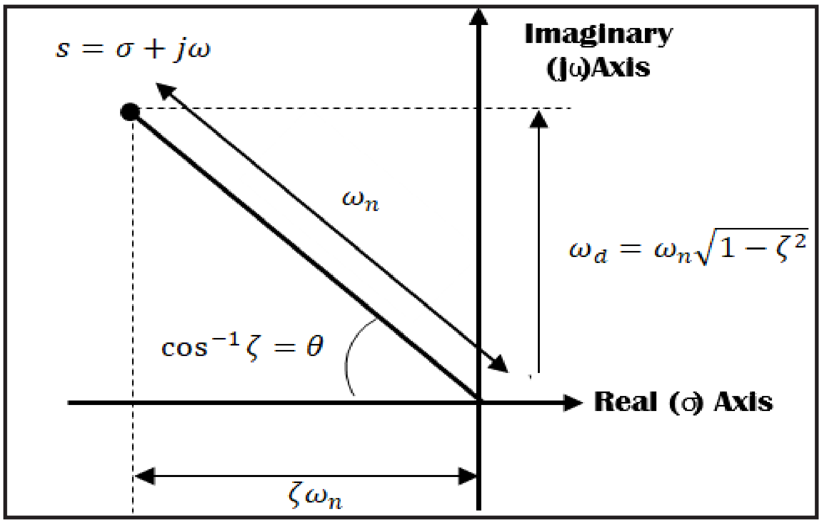

Figure 2-1: Pole Mapping on Second Order System

2.4. Jacobian

The Jacobian method collects the first-order partial derivatives of a multivariate function that can be used for backpropagation. Consider a dynamical system of the form , where is the (component-wise) derivative of x with respect to the evolution parameter t (time), and is differentiable. If , then is a stationary point (steady state). By the Hartman-Grobman theorem, the behavior of the system near a stationary point is related to the eigenvalues of , the Jacobian of F at the stationary point. Specifically, if the eigenvalues all have real parts that are negative, then the system is stable near the stationary point. If any eigenvalue has a real part that is positive, then the point is unstable. If the largest real part of the eigenvalues is zero, the Jacobian matrix does not allow for an evaluation of the stability.

2.5. Trim Condition

To establish a trim equilibrium, the pilot needs to adjust the elevator and thrust to obtain a lift force equal to the weight and forward force equivalent to the drag at the desired speed and flight path angle. To prevent a rotational motion, the resultant moment should also be zero. By considering only the longitudinal motion, the straight or symmetric flight can be applied. For a given aircraft mass, the center of gravity (c.g.) position, altitude, and airspeed, the symmetric trim is described by the aerodynamic operating condition, namely angle of attack, thrust, pitch attitude, elevator angle, and flight path angle. Other operating condition parameters can then be derived as required.

The total axial force is given by resolving the total lift L, total drag D, weight mg, and thrust T into the ox axis and these components must sum to zero in trim. Similarly, the total normal force is given by resolving the forces into the oz axis and these also must sum to zero in trim. The total pitching moment about the c.g. is given by the sum of the wing–body, tailplane, and thrust moments, and these moments must sum to zero in trim. These equations of motion are shown in (2-4) to (2-6).

$$T\cos\tau + L\sin\alpha - D\cos\alpha - mg\sin(\gamma+\alpha) = 0 \quad (2\text{-}4)$$

$$-T\sin\tau + L\cos\alpha - D\sin\alpha - mg\cos(\gamma+\alpha) = 0 \quad (2\text{-}5)$$

$$M_A - Td = 0 \quad (2\text{-}6)$$

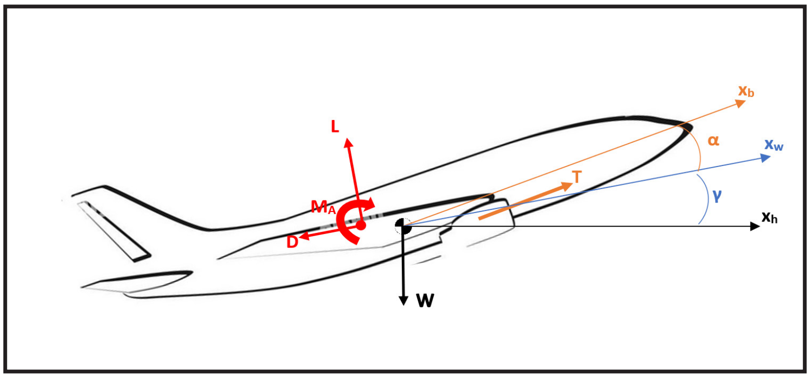

Figure 2-1: Aircraft Free-Body Diagram

For a straight stationary flight, the aerodynamic forces and moment can be described to its component using the Taylor first order as shown in equations (2-7) to (2-10).

$$D = \bar{q}S(C_{D_0} + C_{D_\alpha}\alpha + C_{D_{\delta e}}\delta e) \quad (2\text{-}7)$$

$$L = \bar{q}S(C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\delta e}}\delta e) \quad (2\text{-}8)$$

$$M_A = \bar{q}Sc(C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta e}}\delta e) \quad (2\text{-}9)$$

$$\bar{q} = {}^{1}/_{2}\,\rho V^2 \quad (2\text{-}10)$$

The aerodynamic coefficient C_D, C_L, and C_m can be determined via simulation or wind tunnel test.

3. Longitudinal Control Design

3.1. Design Requirements and Objectives

The document MIL-F-8785c is used as a standard for handling qualities. This document is a military specification for flying qualities of piloted airplanes published by the United State Airforce. According to this document, the handling qualities vary depending on the aircraft class, flight qualities level, and flight phase. Albatross is considered as class I while flying of qualities of level 1 is needed to be clearly adequate for the mission flight phase. The requirements are given in Table 3-1.

Table 3-1: Handling Requirements

Modes	Parameters	Min. Value	Max. Value
Short Period	Damping ratio	0.30	2.00
Phugoid	Damping ratio	0.04	-

3.2. Stability Augmented System Design

The design of the control system is implemented on a trim point that serves the cruise condition as shown in Table 3-2.

Table 3-2: Trim Point Condition

Parameters	Value
Airspeed	17 m/s
Altitude	300 m

The linear state space is constructed with the aircraft parameters around the trim point. The aircraft parameters are given in Table 1-1 and the aerodynamic parameters are obtained by CFD simulation. Then, by using the aerodynamic equation in (2-7) to (2-10) as well as the forces and moments equation which satisfy the trim condition in (2-4) to (2-6) we can get state space as shown in Table 3-3.

Table 3-3: Linear State Space at Trim Point

Matrices	Value
A	$\begin{bmatrix} 0 & 0 & 0 & 1 \\ -9.7655 & -0.1690 & 0.9784 & -0.8740 \\ -0.5220 & -0.6790 & -8.7915 & 16.3511 \\ 0 & 0.1134 & -2.1211 & -7.0935 \end{bmatrix}$
B	$\begin{bmatrix} 0 & 0 \\ 0.1509 & 0.0012 \\ -2.8235 & 0 \\ -62.1149 & 0 \end{bmatrix}$
C	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
D	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

The state, $\bar{\boldsymbol{x}}\boldsymbol{x}$ indexes in order are pitch angle θ, translational speed on the x-axis u, translational speed on the z-axis w, and pitch rate q. Meanwhile, the input, $\bar{u}\bar{u}$ indexes in order are elevator deflection δe and engine rotational speed ω (see eq. 2-1).

The open loop characteristics can be calculated using eq (2-3) and are shown in Table 3-4.

Table 3-4: Open Loop Characteristics

Poles	Damping Ratio	Frequency (rad/s)
-0.0902+0.485i	0.183	0.493
-0.0902-0.485i	0.183	0.493
-7.94+5.86i	0.804	9.87
-7.94-5.86i	0.804	9.87

The first couple of poles with a lower frequency and damping ratio is representing the phugoid mode, while a second couple of poles with a higher frequency and damping ratio represent the short-period mode.

To find the feedback gain, first, the desired damping value should be established. Then, the closed-loop system is analyzed by giving an increment to the feedback gain.

$$K_p = K + \Delta K \tag{3-1}$$

and,

$$K_n = K - \Delta K \tag{3-2}$$

K_p represents gain with a positive increment while K_n represents gain with a negative increment. The error between the damping ζ and the desired damping ζ_{des} can be written as

$$R = \zeta - \zeta_{des} \tag{3-3}$$

If R_n is the damping error of the negative increment gain and R_p is the damping error of the positive increment gain, then the average error R_d can be written as

$$R_d = (R_p - R_n)/(2\,\Delta K) \tag{3-4}$$

The gain feedback can be found iteratively by

$$K^{(k+1)} = K^{(k)} - \frac{R}{R_d} \tag{3-5}$$

Where:

$K^{(k)}$ = k^{th} iteration of gain feedback

$K^{(k+1)}$ = k+1 iteration of gain feedback

This iteration however doesn't work when the damping ratio $\zeta = 1\zeta = 1$ since both positive and negative increments to the gain feedback will not change the value of the damping ratio. In this case, then the next iteration of the gain feedback can be obtained by adding or reducing the gain with the increment directly. The standard convergence condition is when the damping error $R < 0.01$. The algorithm to obtain the gain feedback is given in Figure 3-1.

In case of a high steady-state error response, the integral gain can be incorporated into the gain feedback. Thus, the gain feedback will be formed as a vector containing the proportional gain (K_p) and integral gain (K_I). The value of integral gain can be set as a portion of the proportional gain.

$$K_I = \mathrm{c} * K_p \tag{3-6}$$

Considering the requirement given in Table 1, a desired damping for the short-period mode is set at ζ_{des} = 0.85. Then, by following the algorithm given in Figure 3-1 and formula (3-1)-(3-6), the gain feedback can be found as K = -0.049.

The longitudinal stability augmented system is mainly to improve the short-period response of aircraft. This can be done by feeding the pitch rate value to the elevator input. The block diagram can be seen in Figure 3-2.

4. Simulation

To observe the system response, numerical simulations are conducted using MATLAB/Simulink. The flight mechanical model is made using the values given in Table 3-3, while the gain feedback is found using equations (3-1) to (3-6) and also from the algorithm shown in Figure 3-1. Figure 4-1 shows the natural step response of the UAV to the elevator input of 1 degree for trim condition V = 17 m/s and altitude h = 300 m, while Figure 4-2 shows the response of the UAV with longitudinal stability augmented system implemented with the same parameters.

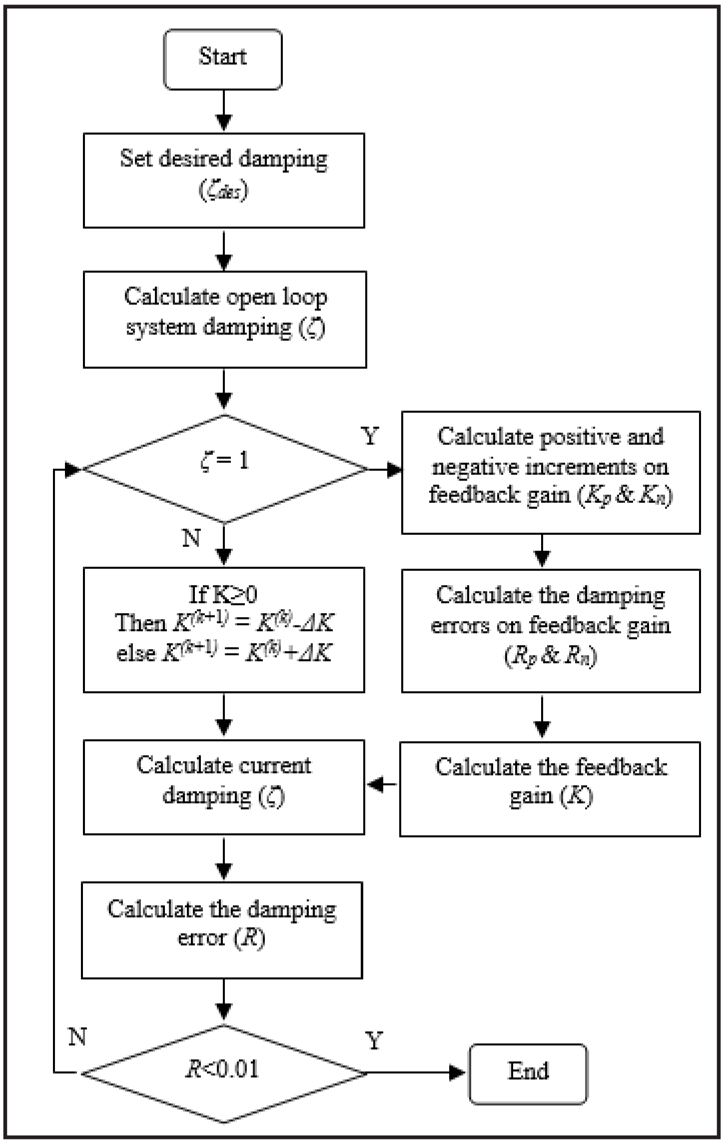

Figure 3-1: Algorithm to Find Gain Feedback

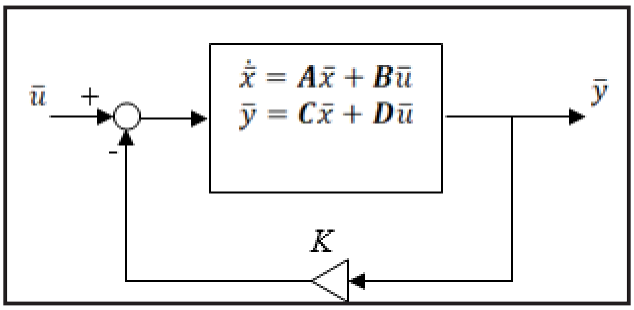

Figure 3-2: Feedback System Block Diagram

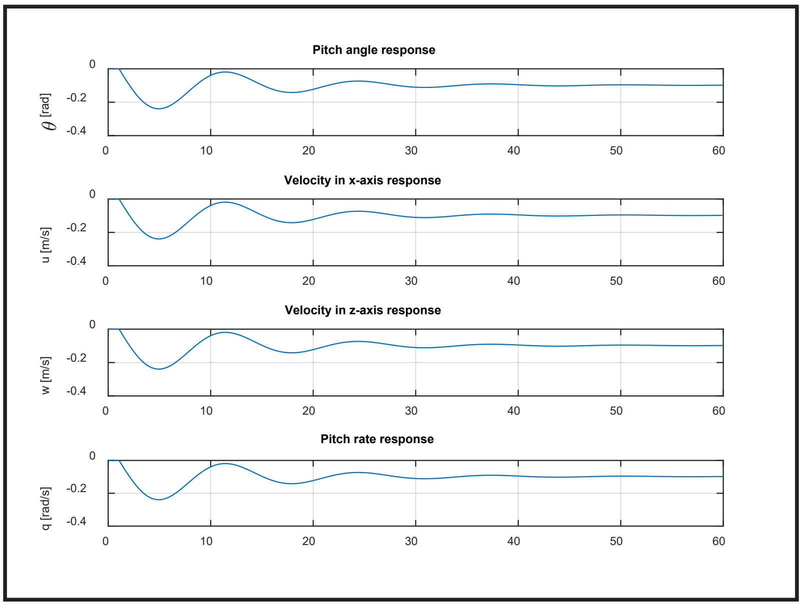

Figure 4-1: Natural Step Response of the UAV to the Elevator Input of 1 Degree for Trim Condition V = 17 m/s

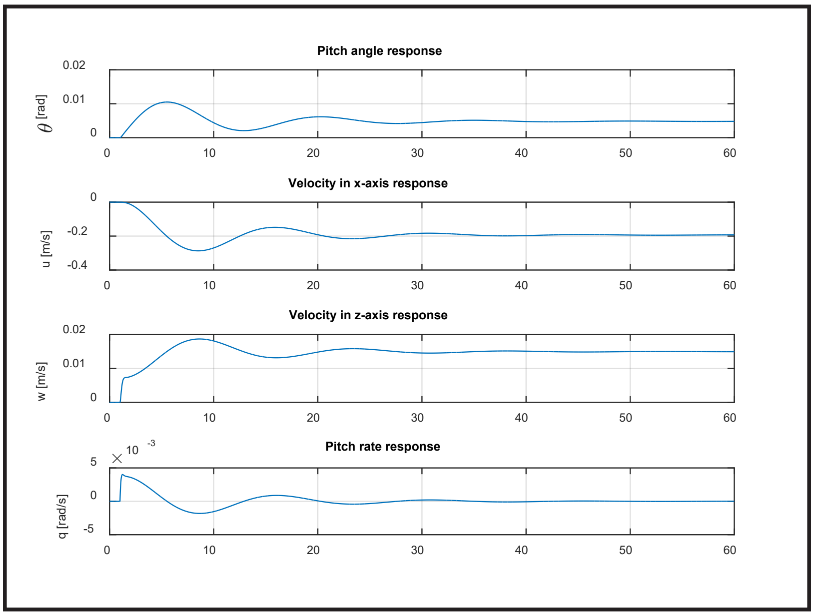

Figure 4-2: Step Response of the UAV with Longitudinal Stability Augmented System

In Figure 4-1, we can see that the UAV is naturally stable in the longitudinal dimension, but the overshoot is relatively high. By adding a longitudinal stability augmented system, the overshoot can be reduced leading to faster response and an increase in UAV stability as shown in Figure 4-2.

5. Design Requirements Verification

The longitudinal stability augmented system that has been designed is verified according to the design requirement and objectives. The verification is shown in Table 5-1.

Table 5-1: System Verification to Design Requirements

Modes	Requirements	Values
Short period	$0.3 < \zeta_{sp} < 2.0$	0.85
Phugoid	$0.04 < \zeta_p$	0.22

As seen in Table 5-1, the system that has been designed conforms with the design requirements and objectives.

6. Performance Comparison

To find out how the control design performance stack up against other methods, another type of control is made and then simulated using the same parameters. Here, the comparison is made using the classical PID control. The gain control (K_P, K_I, and K_D) is found by tuning (trials and errors). The step responses to elevator input for both designs are shown in Figure 6-1 and Table 6-1.

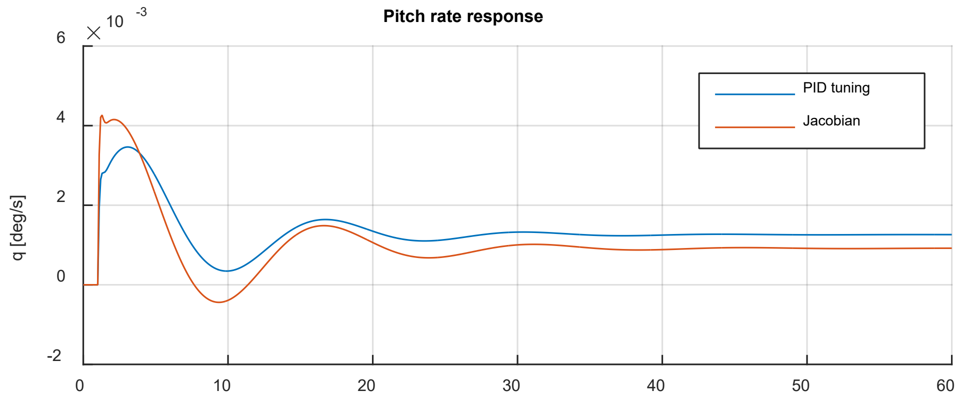

Figure 6-1: UAV Pitch Rate Response to the Elevator Step Input

Table 6-1: Step Response Performance Comparison

Parameters	Jacobian	PID
Gain [K_P K_I K_D]	[-0.0495 -0.0099 0]	[-0.0278 -0.0139 0]
Rise Time (s)	0.023	0.052
Settling Time (s)	33.042	32.245
Overshoot (%)	364	175
Damping Short Period	0.85	0.83
Damping Phugoid	0.22	0.27

From Figure 6-1 and Table 6-1, we can see that in this case, the Jacobian method gives a faster rise time but a slightly longer settling time as well as a higher overshoot. However, the Jacobian method gives damping to the short-period mode which is closer to the desired value (ζ_{des}). It should also be noted that the classical PID method needs several tries before producing gains that give a stable response and satisfy the design requirements criteria. It's also harder for the PID method to guess the gain that gives a damping response near the desired damping value.

7. Conclusions

In this paper, a stability augmentation system for linear control in the longitudinal dimension for UAV Albatross has been designed. The control system is designed in the trim condition on cruising flights at an altitude of 300 m with aircraft speeds of 17 m/s. The system is designed using pitch rate feedback to the elevator input. The designed control system produces a damping ratio of 0.85 for the short-period motion and a damping ratio of 0.22 for the phugoid motion. The value is in accordance with the purpose and design requirements. The longitudinal dimension control system designed is dynamically stable.

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Contributorship Statement

PAPS is the main contributor to this paper.

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