

FEM Modal Analysis of Blades Number Effect in UAV Propellers

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Abstract

The propeller is one of the most important components in an Unmanned Aerial Vehicle (UAV) because it produces the thrust force. However, its structural strength is affected by the vibration phenomenon. The modal analysis can be used to study the dynamic behavior of mechanical structures under dynamic vibration. This study aims to analyze the vibration characteristics of UAV propellers in terms of propeller blade numbers using modal analysis. The propeller configurations used are two, three, and four blades. The Finite Element Method (FEM) is used to calculate the eigen frequencies of the system. The ANSYS Workbench is selected as the FEM software to simulate the study. The result shows that the propeller with 3 blades has the highest eigen frequency compared to the propeller with 2 and 4 blades which could make it preferable for UAV operations.

Keywords: Modal analysis, UAV Propeller, Propeller Blades, Eigen Frequency, Finite Element Method.

1. Introduction

A small Unmanned Aerial Vehicle (UAV) is defined by a weight of fewer than 44 pounds, a speed of less than 100 miles per hour, and operating below a height of 400 ft (Baza & Towhidnejad, 2019). This UAV category become the most popular used in our daily life such as photography, good delivery, news coverage, weather monitoring, etc. In terms of safety, the propeller is one of the most critical components in a UAV. Its function to produce thrust and its high rotational speed becomes the main reason for that consideration. The most common propeller configurations consist of 2 blades, 3 blades, and 4 blades. The rotational speed from the propeller may excite the vibration to other parts of the UAV.

The criticality of a vibrated object can be analyzed with a modal analysis technique. The modal analysis focused on the dynamic behavior of mechanical structures under dynamic excitation. It can determine the dynamic characteristics of a system such as natural frequency, mode shapes, etc. Many studies are using modal analysis to evaluate UAV components such as in its frame (Ahmad et al., 2021; Verbeke & Debruyne, 2016), propeller (Ahmad et al., 2020; Kulandaiyaappan et al., 2021; Raja et al., 2021), wing (Bashir & Rajendran, 2018; Neu et al., 2016; Nikhil A. Khadse & Prof. S. R. Zaveri, 2015) and whole body (Dimitrijević & Kovačević, 2010; Kerschen et al., 2016; Ruseno, 2021).

This study aims to analyze the vibration characteristics of UAV propellers with the same design but different in terms of propeller blade numbers using modal analysis. The result can be used in the selection of UAV components to reduce the negative effect of its vibration characteristic. This report consists of a literature study in Section 2 and the background theory and methodology in Sections 3 and 4. The result and analysis are in Section 5. The last Section is the conclusions and recommendations.

2. Literature Review

This literature study covers 5 publications found which their topic is on modal analysis of UAV propellers. The review focuses mainly on the object of research, the research method used, and the obtained result.

The first 2 publications from (Ahmad et al., 2019, 2020) focused on structural analysis of UAV propellers from the effect of different materials. The first of them investigated the vibration characteristics for three types of material which are Phenolic Epoxy Fiber, Aluminum Alloy, and Carbon Fiber Reinforced Polymer (CFRP). The propeller geometry was not mentioned explicitly but was designed using Creo 2.0 (Ahmad et al., 2019). Similarly, in the second publication, the natural frequencies under static and vibration loading were investigated for two types of material which are carbon fiber-reinforced polymer and glass fiber-reinforced polymer (Ahmad et al., 2020). Both researchers used ANSYS software for FEM calculation, but different versions were used: 16.2 (Ahmad et al., 2019) and 2019 (Ahmad et al., 2020). The result shows that CFRP material has the highest vibration frequency compared to other materials (Ahmad et al., 2019, 2020).

The next 2 publications were authored by (Raja et al., 2021) and (Kulandaiyaappan et al., 2021) also researched modal analysis of UAV propellers with a focus on propeller geometry and material used. The first publication of this group focused on propeller geometry with good pitches for carrying heavy payloads. The propeller sizes used were 10 and 20 inches. Ten different propellers were designed through analytical calculation and modeled in CATIA software (Raja et al., 2021). The second publication used the optimized two propellers having a diameter of 10 inches and 20 inches respectively with pitches of 8 and 20 inches. The various types of lightweight materials effects were analyzed similarly to the first group of publications of (Ahmad et al., 2020). The ten materials used come from three families which are Carbon Fiber Reinforced Polymer Composite, Glass Fiber Reinforced Polymer Composite, and Aluminum Alloy (Kulandaiyaappan et al., 2021). Both research used ANSYS software but different modules: ANSYS FLUENT (Raja et al., 2021) and ANSYS Workbench (Kulandaiyaappan et al., 2021). The result did not mention clearly which design or material was the best but mentioned the selection criteria. The criteria were high thrust production, low reacted deformation, and low induced equivalent stress as a criterion for the best propeller geometry (Raja et al., 2021) and vibrational energies generated for the best propeller material (Kulandaiyaappan et al., 2021).

The last publication focused on reducing the vibration from UAV propellers. It introduced a new vibration-damping technique for UAV propellers (BIELA 24 in diameter and 12 in pitch) utilizing piezoelectric transducers (sensors and actuators). The research conducted experiments and simulation tests for damping performance. It was found from the experiment that the type and location of the piezoelectric transducers concerning the mode shape of the blade mechanical strain affected the damping performance. A finite element modal analysis of a non-rotating propeller without piezoelectric transducers was performed in ANSYS-Workbench 15.0. Then, the numerical results were compared to the experimental measured modal data for verification. The results indicated the vibration could be reduced by applying the transducers to propeller blades at the first mode high modal strain areas (Morad et al., 2015). As a result of this review, none of the research on modal analysis of UAV propellers explored the effect of blade number of propellers. Thus, this niche topic will be explored more in this research.

3. Background Theory

3.1. Multi-degree Freedom system

In general, the system vibration can be considered in many movement directions. It can be approached as a multi-degree freedom system which can be represented by a vibration equation of motion in a matrix form (Shabana, 1997):

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F} \quad (3-1)$$

where:

M - mass matrix

C - damping matrix

K - stiffness matrix

F - forces vector

q - coordinate vector n-degree of freedom

If we assume that the dissipative forces $\mathbf{C}\dot{\mathbf{q}}$ and the external (excitation) forces \mathbf{F} are neglected, the Eq. (3-1) can be simplified into the equation of motion for undamped free vibrations and its matrix form is:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \quad (3-2)$$

Similar to the single degree of freedom system, we can assume that the solution is in the form of:

$$\mathbf{q} = \mathbf{A} \sin(\omega t + \varphi) \quad (3-3)$$

where:

\mathbf{A} - amplitudes vector

ω - natural frequency

φ - phase angle

The differentiation of Eq. (3-3) twice concerning the time and substitute it for Eq. (3-2) leads to the eigenvalue problem:

$$[\mathbf{K} - \omega^2 \mathbf{M}]\mathbf{A} = \mathbf{0} \quad (3-4)$$

This equation has a nontrivial solution if and only if the coefficient matrix is singular, which is:

$$|\mathbf{K} - \omega^2 \mathbf{M}| = 0 \quad (3-5)$$

Eq. (3-5) is called a characteristic equation in the form of a polynomial of order n. The roots of this polynomial denoted as $\omega_1^2, \omega_2^2, \omega_3^2, \dots, \omega_n^2$ are called characteristic values or eigenvalues. Associated with each characteristic values ω_i^2 , there is an n-dimensional vector called characteristic vector or the eigenvector \mathbf{A}_i which can be obtained from Eq. (3-4). The eigenvector (amplitude) \mathbf{A}_i is sometimes referred to as the i-th mode shape, normal mode, or principal mode of vibration.

As a generalization, we may write the general solution of Eq. (3-3) in the form of:

$$\mathbf{q} = \sum_{i=1}^n \alpha_i \mathbf{A}_i \sin(\omega_i t + \varphi_i) \quad (3-6)$$

where α_i and φ_i , $i = 1, 2, 3, \dots, n$ are 2nd arbitrary constants which can be determined from the initial conditions.

$$\mathbf{q}_0 = \sum_{i=1}^n \alpha_i \mathbf{A}_i \quad (3-7)$$

$$\dot{\mathbf{q}}_0 = \sum_{i=1}^n b_i \omega_i \mathbf{A}_i \quad (3-8)$$

which can be written in a matrix form as:

$$\Phi \mathbf{a} = \mathbf{q}_0 \quad (3-9)$$

$$\Phi \omega \mathbf{b} = \dot{\mathbf{q}}_0 \quad (3-10)$$

The matrix Φ , whose columns are the eigenvectors, is called the modal matrix..

3.2. Continuous System

The real object of research such as a propeller in our study can be considered as a continuous system. The Eq. (3-2) for a continuous system can be reformulated based on Newton's second law with the condition for the dynamic equilibrium of the infinitesimal volume become:

$$\rho A \frac{\partial^2 u}{\partial t^2} = \frac{\partial \left(EA \frac{\partial u}{\partial x} \right)}{\partial x} \quad (3-11)$$

where:

ρ - density

A - cross-section area

E - modulus elasticity

u - displacement

x - coordinate axis

t - time

The displacement $u(x,t)$ as the solution of Eq. (3-11) can be assumed to be in the form of:

$$u(x,t) = \left(A_1 \sin\left(\frac{\omega}{c}x\right) + A_2 \cos\left(\frac{\omega}{c}x\right) \right) (B_1 \sin(\omega t) + B_2 \cos(\omega t)) \quad (3-12)$$

Where $A_1, A_2, B_1, B_2, \omega$ are arbitrary constants to be determined by the boundary and initial conditions.

For a propeller that moves on a rotary base, the boundary conditions can be defined into three parts:

- The left end (free condition): moment $u''(0,t) = 0$ and shear force $u'''(0,t) = 0$
- The center (fixed condition): displacement $u(l/2,t) = 0$ and slope $u'(l/2,t) = 0$
- The right end (free condition): moment $u''(l,t) = 0$ and shear force $u'''(l,t) = 0$

Due to the geometry complexity of a real object as a continuous system, the calculation can be performed by numerical methods such as Finite Element Method (FEM).

4. Methodology

The modal analysis in this study is conducted using the Finite Element (FE) Method in ANSYS Workbench software version 19.1. (ANSYS Workbench, 2024). It is a comprehensive simulation environment that incorporates various analysis tools, including structural, thermal, fluid, electromagnetic, and more. It incorporates robust solvers for structural dynamics, making it well-suited for modal analysis. Users can define boundary conditions, apply constraints, and specify excitation or damping conditions easily. This streamlines the process of preparing a model for modal analysis. The software includes automated meshing capabilities that help generate high-quality meshes with minimal user intervention. Proper meshing is crucial for accurate modal analysis results. Also, it facilitates the extraction of eigenvalues and eigenvectors, which are essential in modal analysis.

The Analysis type used in this study is the Modal study. This module allows to conduct modal analysis of any kind of structure geometry. The geometry used is a UAV propeller in three-dimensional (3D) with 316 mm in diameter as shown in (Figure 4-1) (Polak, 2019). This propeller is suitable for small UAVs with a wingspan of around 1 m. It is also the only one available on the internet with three variances for 2, 3, and 4 blades. The airfoil used is MH-112 for the root and MH-121 for the tip.

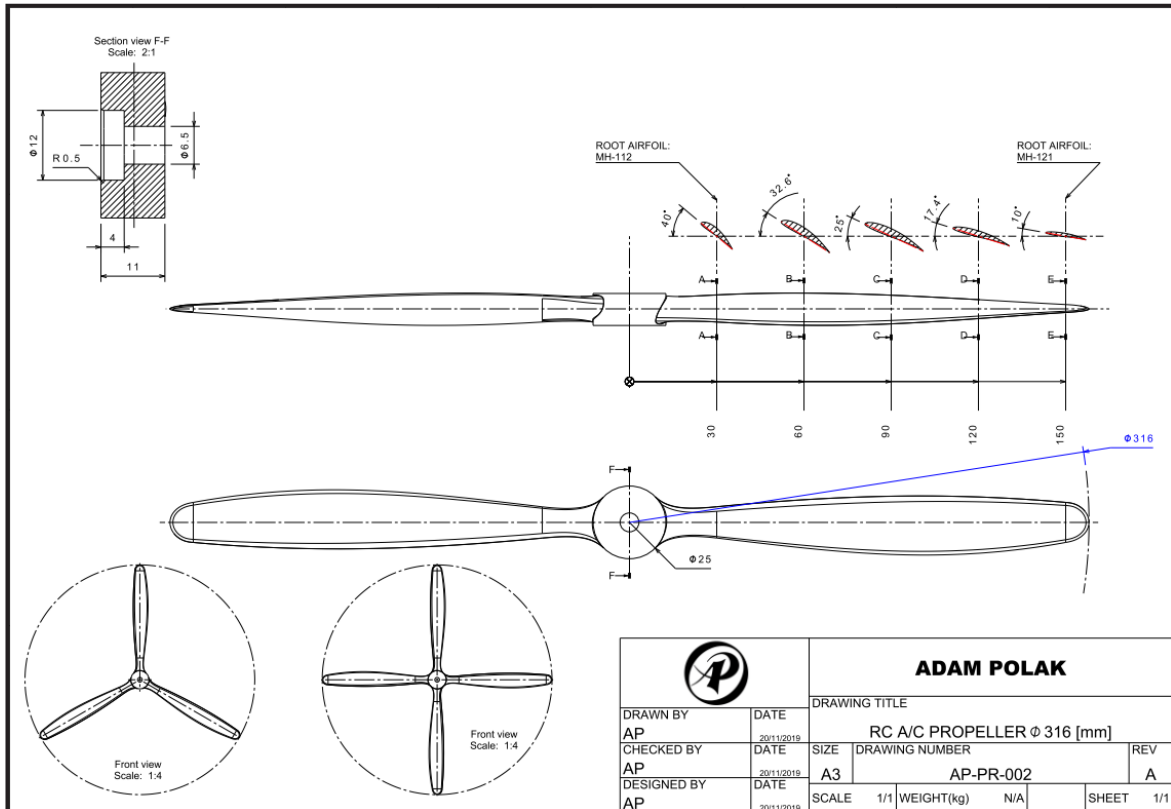


Figure 4-1: Propeller geometry drawing used in this study (Polak, 2019).

The 3D solid in the FE model has the advantage of representing a complex geometry and provides a detailed analysis of stress, strain, and other mechanical behaviors through the entire volume of the structure. However, it requires more computational resources, and the results are sensitive to mesh quality.

The geometry is constructed in CAD application and imported to the ANSYS Workbench software. The material used is Carbon Fiber Reinforce Polymer the properties are available from publications mentioned in the above literature review as shown in (Table 4-1) (Ahmad et al., 2019, 2020).

Table 4-1: Carbon Fiber Reinforced Polymer Properties (Ahmad et al., 2019, 2020)

| Property | Value | Unit |
|-----------------|------------|--------------------|
| Density | 1600 | Kg.m ⁻³ |
| Young’s Modulus | 70000 | MPa |
| Poisson’s Ratio | 0.3 | |
| Bulk Modulus | 5.8333E+10 | Pa |
| Shear Modulus | 2.6923E+10 | Pa |

The complete mesh used in this study is shown in (Figure 4-2) which is created using the automated meshing module of the ANSYS Workbench software. The resulting mesh consists of 8786 nodes and 4068 elements. The mesh size is 0.31719 m in bounding box diameter, 2.0092e-004 m² in average surface area, and 1.1096e-005 m in minimum edge length.

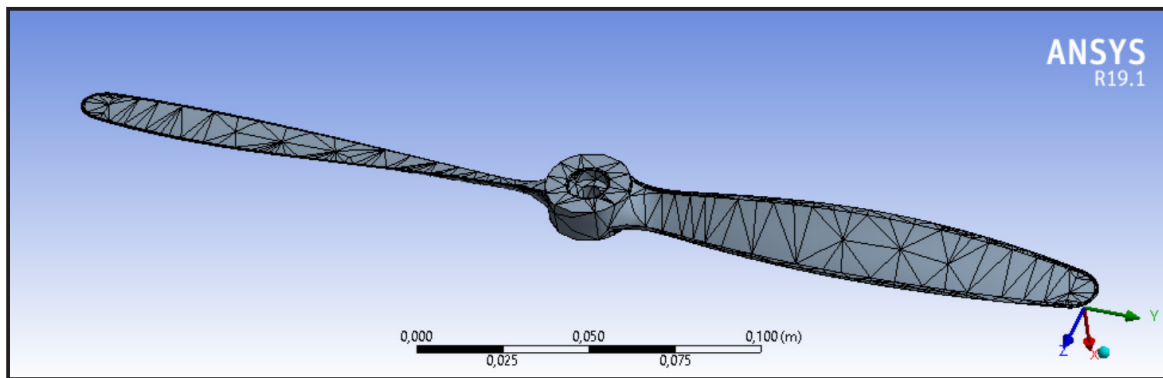


Figure 4-2: Propeller meshing geometry.

For the boundary conditions, a fixed support is applied in the propeller center as shown in (Figure 4-3). The tips of the propeller are left in free condition. The setting of the eigenfrequency study uses a variation number depending on a number of propeller blades with solver Mechanical APDL. For our study, we simulate 3 propeller configuration which consists of 2, 3, and 4 blades.

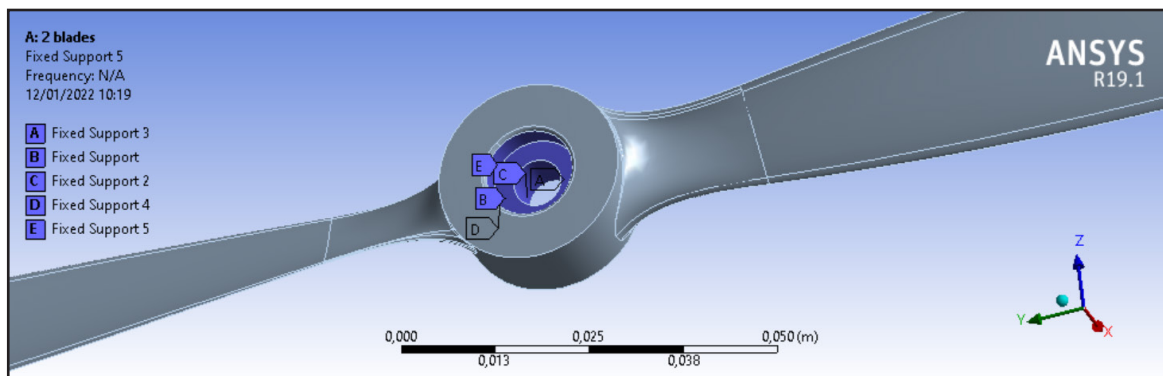


Figure 4-3: Fixed support in the propeller center.

5. Result and Analysis

The result of FEM modal analysis for 2, 3, and 4 blade propellers are presented in Figures 4-4, 4-5, and 4-6 respectively. The number of eigenfrequencies analyzed is different for each propeller type due to the higher degree of freedom in a greater number of propeller blades. It is 6, 9, and 12 eigen frequencies respectively for the 2, 3, and 4 propeller blades number. The colors of the propeller represent the deformation from minimum (blue) to maximum (red).

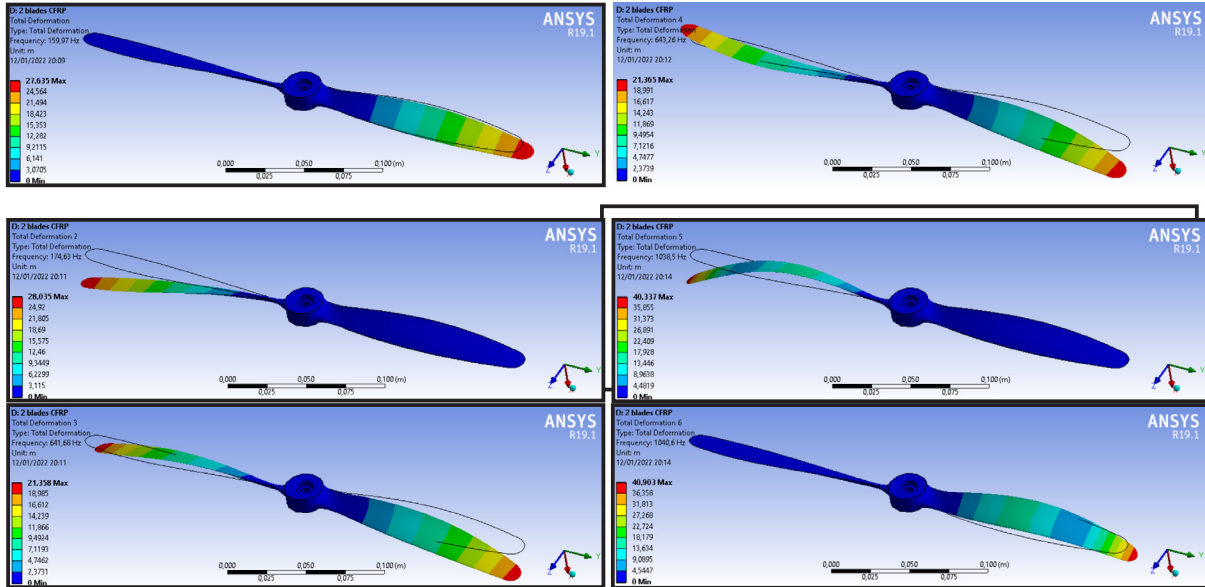


Figure 4-4: FEM Modal Analysis result of two blades propeller.

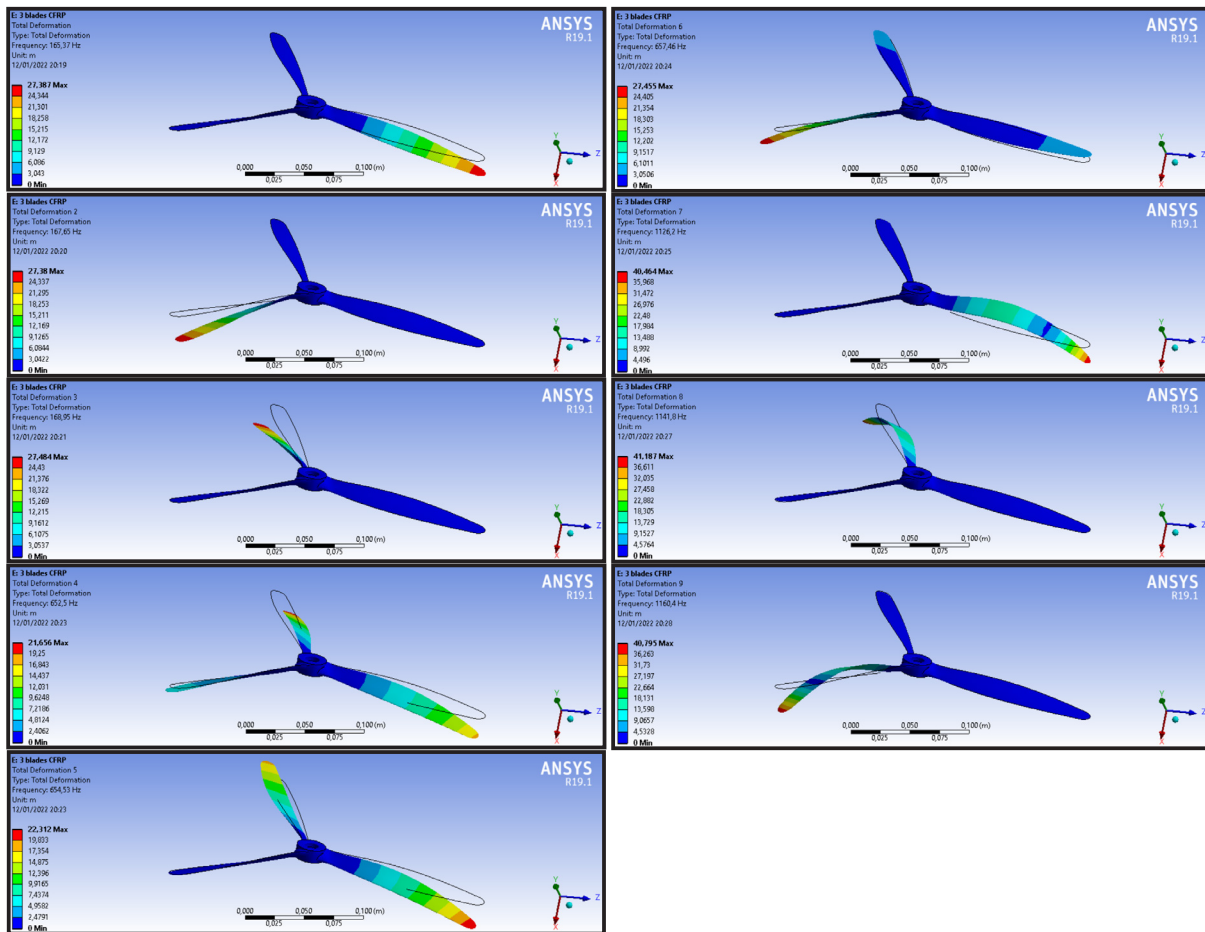


Figure 4-5: FEM Modal Analysis result of three blades propeller.

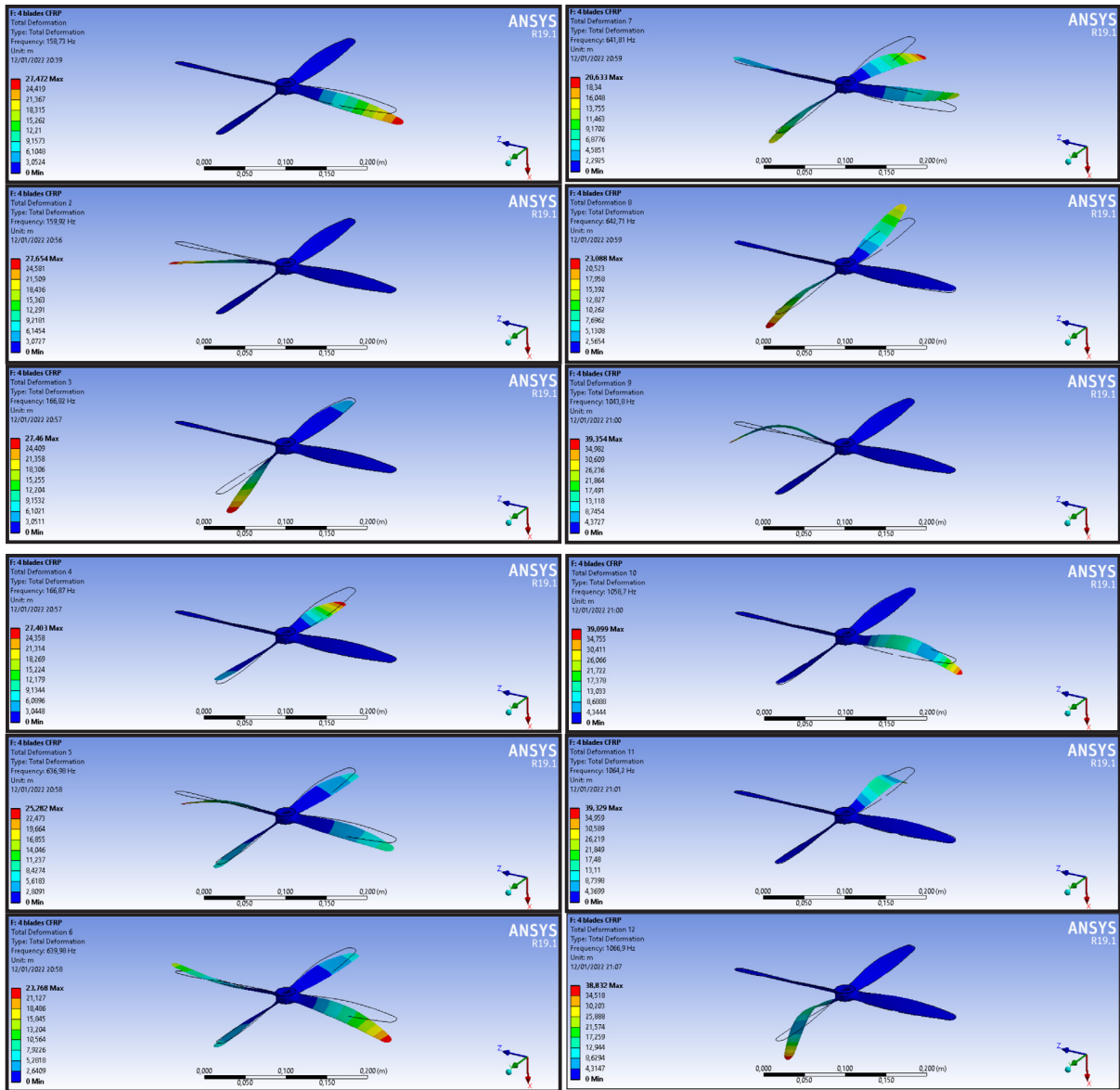


Figure 4-6: FEM Modal Analysis result of four blades propeller.

The resulting eigen frequencies are presented in Table 4-1(a-c) and plotted in Figure 4-7(a). It clearly shows that there are 3 groups of eigen frequency low, medium, and high frequency. This result is similar to the findings from the research of (Ahmad et al., 2020) as shown in Figure 4-7(c) and Table 4-1(d). The low-frequency eigen values are related to the vibration of each blade as shown in Figures 4-4, 4-5, and 4-6. The medium frequency eigen values are related to the vibration of combined blades. Like the low-frequency eigen values, the high-frequency eigen values are related to the vibration of each blade but with higher deformation.

The average eigen frequency for low, medium, and high frequency groups are shown in Figure 4-7(b) which the medium frequencies are around 4 times the low frequencies, and the high frequencies are around 2 times the medium frequencies. It indicates that the propeller with 3 blades has the highest eigen frequency compared to the 2 and 4 blades. This result could be the effect of the shape and distribution of mass along the blades that can impact the natural frequencies and mode shapes. Thus, it is less susceptible to resonance vibration during UAV operations.

The mode shape also affects the magnitude of blade deformation. The maximum blade deformation values for each mode shape are shown in Figure 4-8(a) and the average for each blade number is plotted in Figure 4-8(b). The medium deformations are around 150% of the low deformations, and the high deformations are also around 200% of the medium deforma-

tions. It shows the trend that the propeller with 3 blades has the highest maximum deformation.

Whether a higher eigenvalue indicates better or worse vibration characteristics, depends on the specific requirements and objectives of the application (Tedesco et al., 1999). For UAV propellers, the higher eigenvalues could be desirable to avoid resonance phenomena in UAV operations.

Table 4-1: Eigen Frequencies of Modal Analysis Result: a. two blades, b. three blades, c. four blades, d. two blades from (Ahmad et al., 2020)

| (a) | | (b) | |
|------|----------------|------|----------------|
| Mode | Frequency [Hz] | Mode | Frequency [Hz] |
| 1 | 159.97 | 1 | 165.37 |
| 2 | 174.63 | 2 | 167.65 |
| 3 | 641.68 | 3 | 168.95 |
| 4 | 643.26 | 4 | 652.5 |
| 5 | 1038.5 | 5 | 654.53 |
| 6 | 1040.6 | 6 | 657.46 |
| | | 7 | 1126.2 |
| | | 8 | 1141.8 |
| | | 9 | 1160.4 |

| (c) | | (d) | |
|------|----------------|------|----------------|
| Mode | Frequency [Hz] | Mode | Frequency [Hz] |
| 1 | 158.73 | 1 | 183.23 |
| 2 | 159.92 | 2 | 186.92 |
| 3 | 166.82 | 3 | 545.06 |
| 4 | 166.87 | 4 | 581.27 |
| 5 | 636.98 | 5 | 1149.3 |
| 6 | 639.98 | 6 | 1168.9 |
| 7 | 641.81 | | |
| 8 | 642.71 | | |
| 9 | 1043.8 | | |
| 10 | 1058.7 | | |
| 11 | 1064.2 | | |
| 12 | 1066.9 | | |

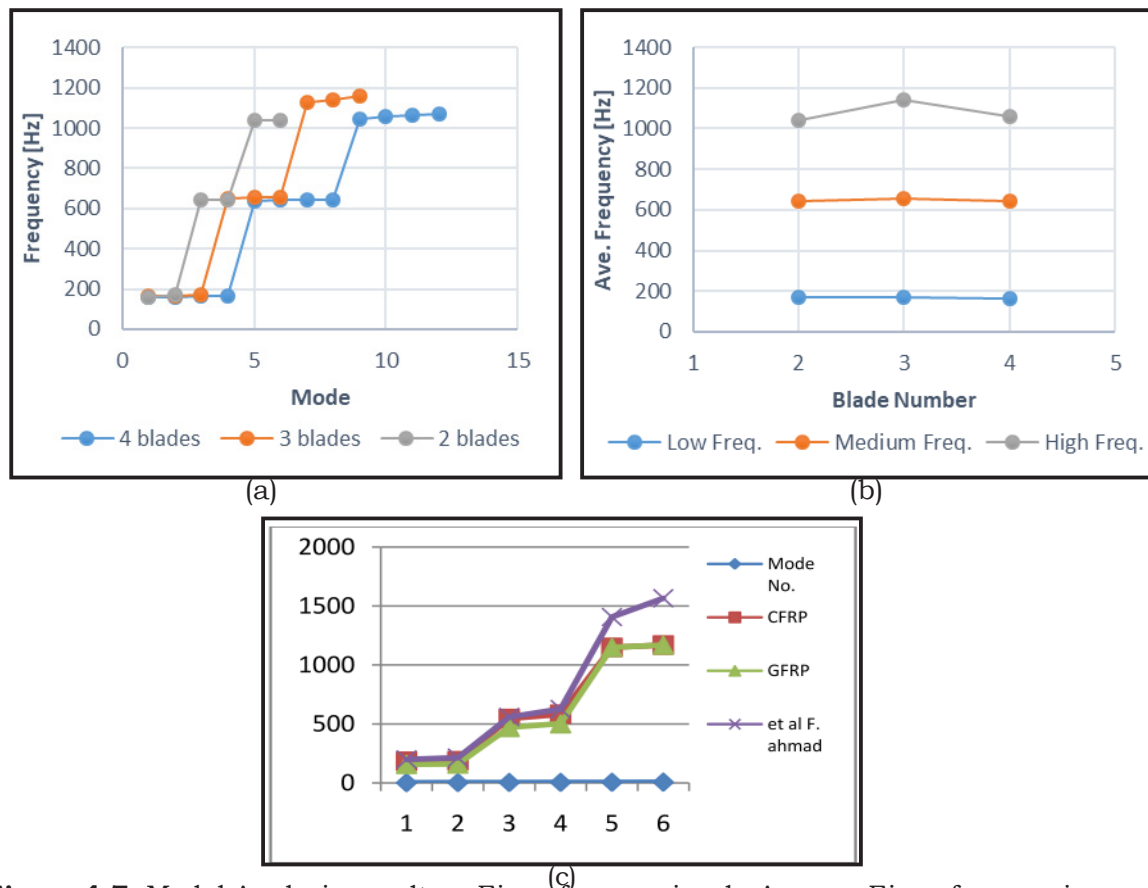


Figure 4-7: Modal Analysis result: a. Eigen frequencies, b. Average Eigen frequencies, c. Eigen frequencies of the 2-blade propeller from (Ahmad et al., 2020).

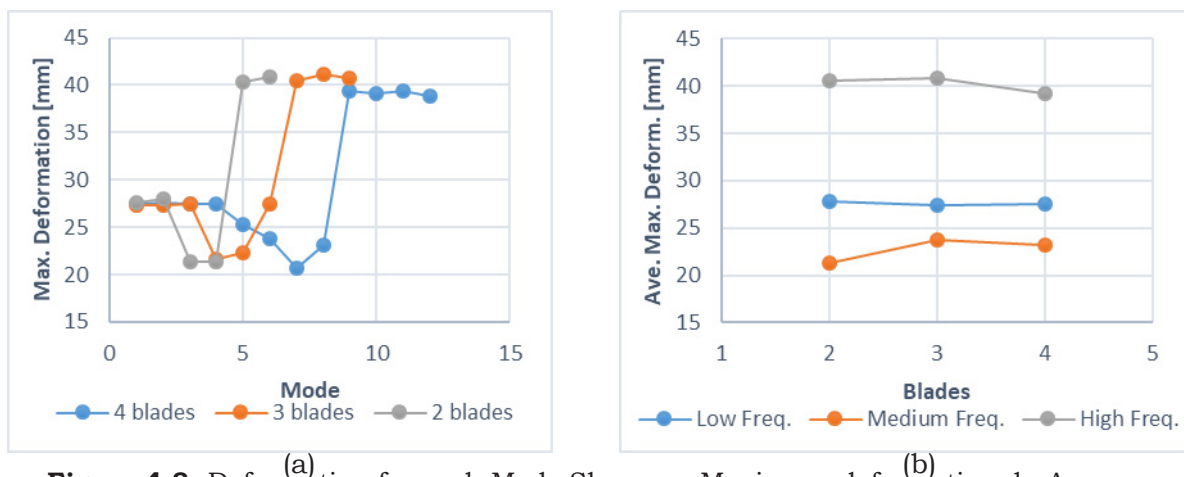


Figure 4-8: Deformation for each Mode Shape: a. Maximum deformation, b. Average maximum deformation.

As a comparison, we found one publication that uses modal analysis to evaluate the effect of the number of propeller blades in the ship. It conducted an experimental simulation of modal analysis for ship propellers with 2, 3, and 4 blades. The result showed that the 2-blade propeller has the highest natural frequencies for the first 6 modes as shown in Figure 4-9 (Abdullah et al., 2020). Our result seems to be different from their finding.

There are several different conditions between these two studies as shown in Table 4-2. Since the modal analysis mainly depends on the geometry and material properties, these differences are made possible. However, this discrepancy should be explored more in further research.

Table 4-2: Condition comparison between our UAV propeller and Ship propeller research

| Condition | Our UAV Propeller | Ship Propeller (Abdullah et al., 2020) |
|---------------------|---------------------------------|---|
| Application | Flying in the air | Propel in the water |
| Geometry | Slim blade | Round blade |
| Material | Carbon Fiber Reinforced Polymer | Steel |
| Simulation Software | ANSYS Workbench 19.1 | Nastran/Patran |

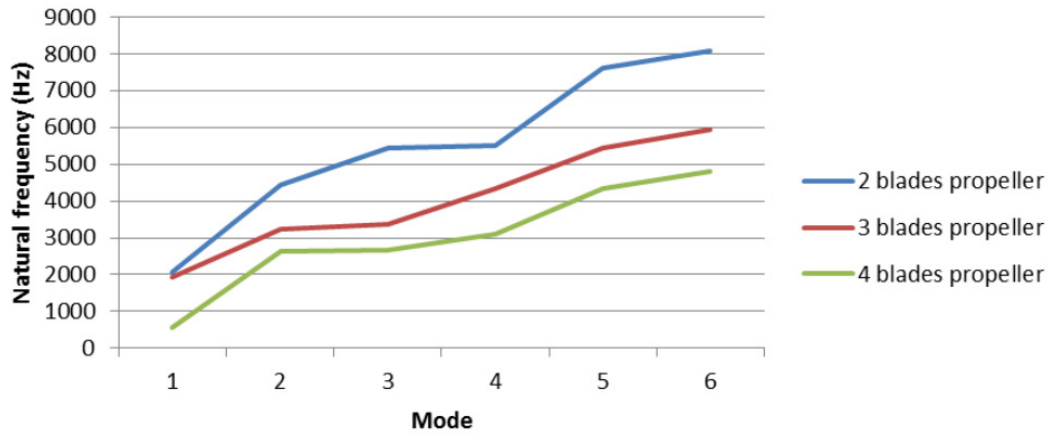


Figure 4-9: The natural frequency of ship propellers with different blade numbers (Abdullah et al., 2020).

6. Conclusions and Recommendations

This study aims to analyze the vibration characteristics of UAV propellers with the same design but different in terms of propeller blade numbers using modal analysis. According to the result and findings from the FEM analysis using the ANSYS Workbench application, the study concludes that:

- There are three eigen frequency categories for each propeller type which are low, medium, and high frequencies. The low-frequency group of eigen values is related to the vibration of each blade. The medium frequency group of eigen values is related to the vibration of combined blades. The high-frequency group of eigen values is related to the vibration of each blade but with higher deformation compared with the lowest group.
- The UAV propeller with 3 blades has the highest eigen frequency compared to the propeller with 2 and 4 blades. For UAV propellers, the higher eigenvalues could be desirable to avoid resonance phenomena and improve the safety of UAV operations.

It recommends that these findings should be confirmed in experiment result with larger variation of propeller geometry and material, and also with considering the engine’s natural frequency effect.

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