Wave: Jurnal Ilmiah Teknologi Maritim Volume 14 Nomor 1, Juli 2020: Hal: 19-26

PENGEMBANGAN MODEL STRUKTURAL ANALITIK UNTUK *GRILLAGE* KOMPOSIT

Development of Analytical Structural Model for Composite Grillage

Dwitya Harits Waskito[1] dan Adam Sobey[2]

[1]Student of University of Southampton, Southampton
[2]Lecturer of University of Southampton, Southampton

Email: dwitya.harits@bppt.go.id

Diterima: 5 Mei 2020; Direvisi: 29 Juni 2020; Disetujui: 30 Juni 2020

Abstrak

Dengan permintaan dan pertumbuhan pasar yang tinggi untuk pembuatan perahu yang terbuat dari komposit, terdapat sebuah kompetisi untuk mendesain sebuah kapal dengan kualitas baik dan dengan material yang lebih ringan, dimana hal tersebut menyebabkan ketebalan struktur dari perahu akan semakin tipis. Untuk mencapai tujuan tersebut dibutuhkan perhitungan pada struktur komposit yang akurat dan cepat. Pada struktur kapal, konfigurasi yang umum pada penegar adalah menggunakan tipe *grillage*. Akan tetapi sebagian besar metode dalam perhitungan *grillage* diperuntukkan untuk struktur dengan material besi. Oleh karena itu, terdapat perbedaan antara teori dari *grillage* dengan material besi dan komposit di mana teori dari *grillage* tersebut akan dimodifikasi, terutama dari perhitungan defleksi struktur tersebut. Terdapat dua metode umum yang digunakan pada analisa ini yaitu teori Navier Grillage sebagai metode analitis dan metode elemen hingga sebagai pembanding. Dua jenis komposit (karbon dan *e-glass*) dengan nilai perbedaan yang besar pada *modulus young* dianalisa dan hasil menyatakan bahwa terdapat perbedaan yang signifikan antara kedua metode tersebut nilai perbedaan sebesar 12% (4.46 mm) untuk karbon *glass* dan 26% (26.37 mm) untuk *e-glass*. Untuk menganalisa perbedaan tersebut, hasil dari defleksi telah dianalisa dan dibandingkan dengan nilai dari properti elastis komposit untuk material *e-glass.* Persamaan empiris didapatkan dengan mengembangkan hubungan antara grafik defleksi pada kedua metode dan properti pada komposit seperti nilai E_1 dan E_2 dimana persamaan empiris tersebut dapat meningkatkan akurasi perhitungan pada teori Navier Grillage untuk *grillage* komposit pada material *e-glass*.

Kata kunci: *grillage*, komposit, struktur, defleksi, *e-glass*

Abstract

With the high demand for composite boats, there is a competition to design a high-quality ship which leads to thinner boat's hull structure. To meet this objective, a highly accurate and computationally fast calculation is needed. In ship structure, the most used configuration of the stiffeners is by using grillage. There is a gap between steel and composites for grillage theory which will be modified especially for the deflection. There are two methods used in this analysis, Navier Grillage theory as the analytical method and finite element analysis as the benchmark tools. To develop analytical method of composite *grillages, the current method was investigated by comparing the analytical results and FEA. Two composites with high difference in Young Modulus were analyzed and the results shows that there is significant difference of results with two previous method. The differences are 12 % (4.46 mm) for carbon glass and 26 % (26.37 mm) for e-glass. The deflection results of two method were analyzed with every composite elastic properties of e-glass. Empirical equation was developed from the relation between deflection graph of two methods and composite properties to increase the accuracy of Navier Grillage theory for e-glass composites grillage.*

Keywords: *grillage, composite, structure deflection, e-glass*

INTRODUCTION

Several industries such as marine, aeronautical, and civil use composite as the main material due to the excellent strength to the weight characteristics, high corrosion resistance, flexible structural properties, and good accessibility of repair and maintenance. With the mass production of the high-speed composite boat and the competition amongst competitor, the boat builders and engineers need to adjust the design to the lower thickness of composite material to reduce the boat's weight and subsequently to increase speed. Moreover, they also need to maintain the structural performance such as stress, bending moment and deflection of the boat's stiffened bottom plate with the thinner material.

In the structural modeling, stiffened plate is used due to its simple configuration and easy to fabricate. The most typical configuration of the stiffened plate in the marine industry is a grillage. The term grillage means the intersecting structural beams which are normally loaded to the surface (Clarkson, 1965).

Time by time, the structural configuration of stiffened plate is becoming more complex with the increasing number of the girders and stiffeners, the different spacing of the stiffeners, more complicated shape (curvature, square, rounded) and the increasing number of the layer in the composite material. With this improving difficulty in the computational of the grillage, the process can be time-consuming and leads to the computational expense. Therefore, the more accurate and efficient simulation and computational method for calculating the structural properties of the composite stiffened plate is necessarily required.

The improvement and modification of the composite stiffened plates already performed by several authors (Maneepan et al., 2007)(Sobey et al., 2008)(Blake et al., 2009)(Sobey et al., 2013). The improvement is based on Navier-Energy Method Grillage theory which uses the Navier-Energy method to calculate the deflection of longitudinal girders and beams which assessed by improving the general algorithm, failure criteria, and reliability methods of composite grillage structures to obtain more accurate results (Vedeler, 1945). The stress and bending moment analysis of Navier Grillage composite compared to FEA with introducing the CLPT (Classical Laminate Plate Theory) to calculate layer by layer stress in the stiffeners with the non-conservative approach (Blanchard et al., 2017). However, in terms of deflection, the detailed and in-depth analysis has not been performed such as the difference between Navier Grillage and FEA, the deflection in the interconnection between the stiffeners, the parametric studies of plate and stiffeners geometry and the investigation of different composite materials to validate Navier Grillage Theory. Moreover, most of the grillage theory is designated for the steel structures with the high value of Young Modulus. Therefore, there are some questions whether the grillage theory also applicable for composite structures with lower Young Modulus and whether the Navier Grillage can be used in composite structure or some improvements and developments need to be done to integrate the Navier Grillage into the composite materials.

Since there is a gap between study of Navier Grillage theory and composite materials, the novelty of this paper is performing the deflection analysis of composite with several parametric and material studies, shape analysis, and development of the Navier Grillage theory by generating an empirical equation to improve accuracy of Navier Grillage code applied into several composite materials.

METHODOLOGY

Composite Grillage

With terms grillage as the structural intersecting beam in a plate, the plate consists of two directional stiffeners, which are longitudinal and transverse. In this study, the terms grillage number means the number of longitudinal and transverse stiffeners. Otherwise, the grillage number also can be said as *r* x *p*. where *r* denotes the longitudinal stiffeners and *p* denotes the transverse stiffeners.

Figure 1. Grillage configuration in ship structures (Simonetta, 2019)

Most of the steel ship structure using open profile such as I-beam, flat bar and angle bar as the stiffeners (Figure 1). These profiles are welded to the plate to provide the longitudinal strength to the ship. However, that profile is not suitable for the small ships such as a yacht, and patrol boat because smaller ships need to reduce structural weight to maintain the speed. The solution of this problem is by using the closed section stiffeners such as top-hat stiffeners. Top-hat stiffener is used due to its good torsional resistance and high bending stiffness resistance (Raju et al., 2012).

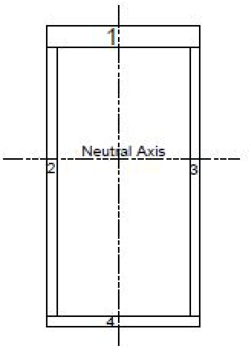

Figure 2. Section of top-hat stiffener

Since the composite structure is analyzed, the top-hat section can represent a difference of elastic properties between the top layer, lower layer in the crown, web and the bottom plate (Figure 2). With the top-hat stiffeners, the spacing between stiffeners can be increased, and ship's outfitting such as cable and small pipes can be inserted in the closed section.

As a preliminary study, the geometry of the plate is determined from previous study with the 3810 mm length and 3810 mm breadth. In this basic case, a square plate was used to simplify the problem. The load (*P*) for the plate is uniform 137.9 kPa applied in an upward direction. The number of stiffeners is ranging from two until five for both longitudinal and transverse stiffeners.

Two common composite materials such as carbon and E-glass were used and the stiffeners type are the top-hat stiffeners. The crown of the stiffeners consists of ten plies with [0 90 0 90 0] s. Two webs and bottom plate consist of 8 layers with [0 90 0 90] s. The thickness of the top is 18.288 mm so each layer has 1.8288 mm thickness, and the thickness of webs and plate is 9.144 mm which means each layer has 1.143 mm thickness. The elastic properties of carbon epoxy are stated as the Table 1 below:

Table 1. Elastic properties of carbon epoxy

No.	Properties	Value
1	E_1 (GPa)	172.4
2	E_2 (GPa)	6.9
3	v_{12}	0.25
4	G_{12} (GPa)	3.45

Sumber: Yang, et al. (2013)

Where E_1 is Longitudinal Young Modulus, E_2 is Transverse Young Modulus, v_{12} is the Poisson ratio and G_{12} is the Shear Modulus. These values are important to the modification of Navier Grillage for composites material since the material properties will be analyzed.

Equivalent Elastic Constant

Since the material of stiffeners is a fiber-reinforced composite which has laminated material, each section could have different elastic properties and each ply also has different mechanical properties. With that case, the laminate stiffness will be different for the crown of the stiffeners and with the plate and width as mentioned above. The output of this calculation is to determine the membrane mode of equivalent elastic properties of each section (E_x) and will be used in Navier Grillage Theory. The process is following the equation of composite stiffness (Datoo, 1991).

Navier Grillage Theory

Navier-Energy method is based on Navier summation of the deflection in the intersection of the grillage. To determine the deflection, bending moments, shear forces, and maximum stress in the composite grillage, for the first approximation, the equation from the stiffened plate used. The stiffeners are assumed as a beam. The deflection at any point can be expressed by summation of trigonometric series.

$$w(x,y):\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}a_{mn}\sin\frac{m\pi x}{L}\sin\frac{n\pi y}{B} \tag{1}$$

a_{mn} is the amplitude coefficient and can be defined by equating the minimum potential energy by the deflection and the integration of the work done by uniform pressure load in the grillage structure. m and n are the odd wave number, and the maximum value is 17 to increase the accuracy of result. L and B are the length and breadth of the grillage.

$$V=\int_0^L\frac{D_r}{2}\left(\frac{\partial^2 w}{\partial x^2}\right)^2_{y=y_p}dx+\int_0^B\frac{D_p}{2}\left(\frac{\partial^2 w}{\partial y^2}\right)^2_{x=x_q}dy \tag{2}$$

The minimum potential energy in the grillage ($\partial V/\partial a_{mn}$) is equal to the work done which specified by:

$$W=\int_0^L\int_0^B P\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}a_{mn}\sin\frac{m\pi x}{L}\sin\frac{n\pi y}{B} \tag{3}$$

The coefficient a_{mn} can be determined by:

$$a_{mn}=\frac{16PLB}{\pi^6 mn\left\{m^4(r+1)\frac{D_r}{L^3}+n^4(p+1)\frac{D_p}{B^3}\right\}} \tag{4}$$

Thus, the a_{mn} value can be substituted into equation 1 Hence, the deflection of the stiffeners can be obtained with:

$$w(x,y):\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{16PLB}{\pi^6 mnD_{total}}\frac{1}{\frac{m^4(r+1)}{L^3}+\frac{n^4(p+1)}{B^3}}\right)\sin\frac{m\pi x}{L}\sin\frac{n\pi y}{B} \tag{5}$$

Finite Element Analysis

Finite element analysis provides the validation and the benchmark tools to analytical method. With the finite element, the physique of the problem such as structural failure can be viewed easily. In this analysis, the finite element analysis was performed using ABAQUS program due to compatibility to input the laminate properties of the composite material. In the modeling process, most of the work was done by using the developed python script (Mutlu et al., 2016).

The first step to model the composite grillages is to model the parts one by one. There are three main parts in this model; longitudinal stiffeners, transverse stiffeners, and plate. The shell element S4R (quadrilateral shell element) was used in this model.

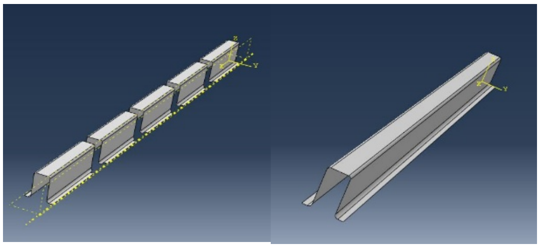

Figure 3. (Left) Transverse stiffener; (Right) Longitudinal stiffener

Figure 3 shows the difference modelling for both stiffeners. For the longitudinal stiffener, the extrusion length can be done for the full length due to the continuity of its configuration. However, for the transverse stiffener, there are some separations due to the interconnection between the longitudinal and transverse stiffener. The length of separations is the width of the longitudinal stiffeners, and the number of separations is determined from the number of longitudinal stiffeners.

In this analysis, fifteen seeds were used which was smaller than previous model with thirty seeds. By using less mesh seed, the number of nodes will be increased with the intention to get a better result.

The processing phase starts by defining the steps of analysis. Step option was used to determine the type analysis. In this analysis, the nonlinear geometry (NLgeom) is used. Step also can determine the increment number and sets the increment size from minimum to the maximum value. After the steps are defined, the type of analysis is needed to be assigned which is static structural analysis. The next step is to apply a load to the model, the type of the load is static pressure load 0.1379 kPa acting upward. After that, the boundary condition needs to be applied. In this analysis, the simply supported analysis is used in the model. The value of U_3 (vertical displacement) is constrained as 0 in the edge of the plate and the flange of the stiffeners which located at the edge of the plate.

RESULT AND DISCUSSION

Validation of MATLAB and FEA

To increase the validity of the MATLAB code of Navier Grillage, the results are also compared with several authors who performed an analysis of composite structures with the similar geometries, load, and topology of the grillages.

Table 2. Validation of MATLAB code for grillage's deflection

	Navier Grillage	(Blanchard et al., 2017)	% error
Grillage Number	δ (mm)	δ (mm)	δ
2	30.40	30.42	0.058
3	30.00	30.02	0.058
4	21.81	21.82	0.045
5	20.03	20.01	0.076

Table 2 shows that all the deflection result of Navier Grillage code is pretty similar to the result of Blanchard's. With that results, it can be concluded that the MATLAB Navier Grillage code is valid because of the very small difference with the result from Blanchard's. Therefore, the MATLAB Navier Grillage code can be used in the analysis afterwards.

Table 3. Validation of FEA model for grillage's deflection

	Navier Grillage	(Upadhya & Loughlan, 1981)	% error
Grillage Number	δ (mm)	δ (mm)	δ
2	34.86	34.69	0.47
3	32.85	32.63	0.69
4	24.05	24.4	1.4
5	21.65	21.51	0.66

Table 3 shows that the maximum error between the modified FEA and Mutlu's FEA model is 1.4%. As expected, the difference is caused by the change of mesh size and by different approach on the height of the stiffeners that were modeled. The FEA Model used 254.84 mm for the stiffener height compared to 254 mm on Mutlu's FEA Model. Since the difference is considered small, the FEA model can be used for the analysis afterwards.

Parametric Studies of E-Glass and Carbon Grillages

Figure 4 shows that there are differences in value between Navier Grillage and FEA. The maximum difference between two methods for carbon composites and E-glass composites is 12% (4.46 mm) and 26% (26.37 mm) respectively. There are some reasons for the differences. Firstly, according to equation 1, the deflection shape is assumed as a half sine wave curve. To prove the assumption, in-depth analysis of deflection shape along the plate length must be conducted to see how the stiffeners deflected. Secondly, the topology of the grillage and the stiffeners properties also can be the cause of the differences. Despite the differences, the results from FEA and Navier Grillage has similar trends which stated that the Navier Grillage theory is currently satisfied and matching with the FEA results for composite grillages for both materials. Since E-glass has bigger difference of deflection result between two methods, and commonly used in boat industry, E-Glass composite is used for in-depth analysis to develop grillage equation.

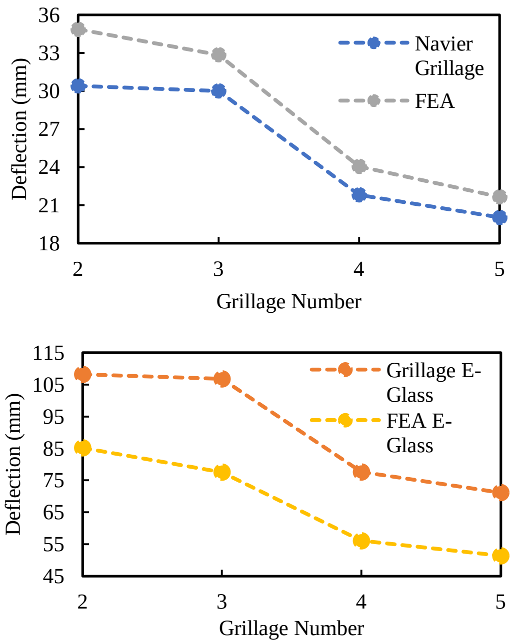

Figure 4. Deflection results of carbon and e-glass composite grillages; (Above) Carbon glass; (Below) E-glass

Deflection Shape Analysis of E-Glass Grillage

The first In-depth analysis of the grillage will be the deflection shape of the composite grillages. According to equation 1, the deflected shape of the grillages is assumed as a half sine wave curve, and it still needs to be proven by the comparison with the deflected shape from FEA for Transverse stiffeners and Longitudinal Stiffeners.

The deflection shape of transverse and longitudinal stiffeners by FEA is shown by Figure 5. If it is integrated, will produce the same amount of area under the curve. Even though there are several ramps occurred in the interconnection of the transverse stiffener, the difference between longitudinal and transverse is not significantly large. Which means that the deflection shape is not a considerable factor of the difference between FEA and Navier Grillage for composite grillages. Since the material properties are the determining factor of differences, the analysis was expanded to several material properties of E-glass to see whether the difference between two methods affected on every E-glass.

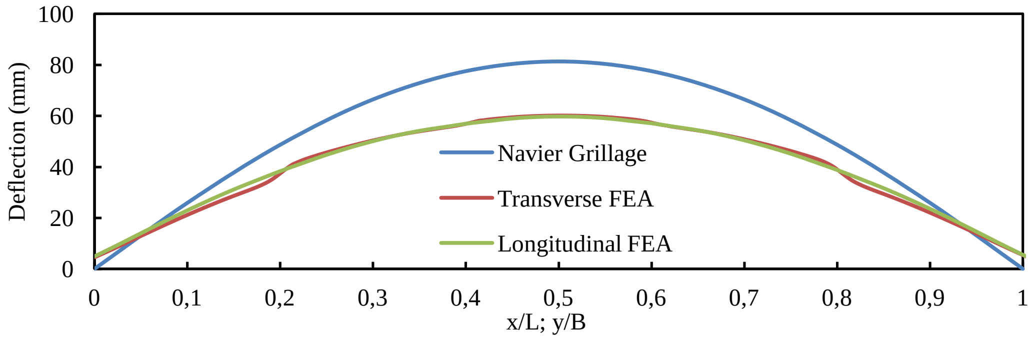

Figure 5. Comparison of deflection shape between Navier Grillage code, longitudinal stiffeners and transverse stiffener

Table 4. Elastic properties of e-glass composite

	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	v_{12}	E_x (GPa)	E_1/E_2	δ Grillage (mm)	δ FEA (mm)
E-Glass 1 (Upadhya & Loughlan, 1981)	30	6	5	0.33	20.60	5.00	151.02	102.96
E-Glass 2 (Biswal et al., 2017)	39	8.6	3.8	0.28	27.08	4.53	114.75	85.29
E-Glass 3 (Performance Composites, 2009)	40	8	4	0.35	27.50	5.00	113.12	83.07
E-Glass 4 (Johnson & Sims, 1983)	40	10	4	0.3	28.22	4.00	109.43	80.99
E-Glass 5 (Blanchard et al., 2017)	43	8	4	0.28	29.10	5.38	106.76	80.11
E-Glass 6 (Sudheer et al., 2015)	45	12	5.5	0.28	32.02	3.75	96.24	69.14
E-Glass 7(Zhang & Matthews, 1983)	53.8	17.9	8.9	0.25	39.64	3.01	77.08	53.87
E-Glass 8(Zhang & Matthews, 1983)	57	10.32	4.014	0.23	38.50	5.52	81.02	63.89

Material Analysis of E-Glass Grillage

The second in-depth analysis of the grillage is done to know the deflection results of several material with different properties. The analysis of E-glass composites grillages was taken using 4x4 grillages. Table 4 shows that commonly, the higher value of E_1 (Longitudinal Young Modulus) resulting the deflection to be smaller. However, to find the relation between the elastic properties and deflection, the ratio of two elastic properties (E_1 and E_2) is plotted against the deflection for both method; FEA and Navier Grillage. With the same deflection trend for both FEA and Navier Grillage, the Figure 6 and 7 can be used for determining the empirical equation for E-Glass composite grillages, by assessing the amount of difference between deflection of FEA and Navier Grillage.

From Graph 7, the trend for both Navier Grillage and FEA is similar and the relation between two different deflection results can be formulated. Each deflection graph has its equation and the factor can be determined by dividing the graph equation of FEA over the equation of Navier Grillage. Empirical equation of deflection for E-Glass composite grillages can be shown as:

$$w_{E-Glass} : \frac{5.906\left(\frac{E_1}{E_2}\right)+47.45}{5.074\left(\frac{E_1}{E_2}\right)+77.17} w(x,y) \qquad (6)$$

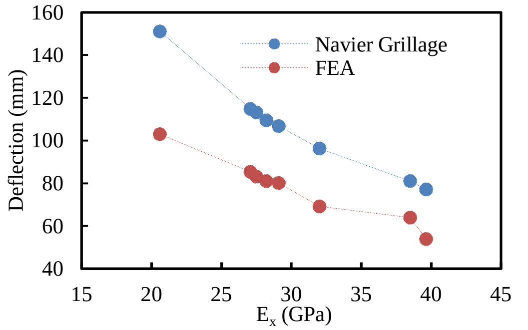

Figure 6. Deflection of composite grillages plotted against Ex

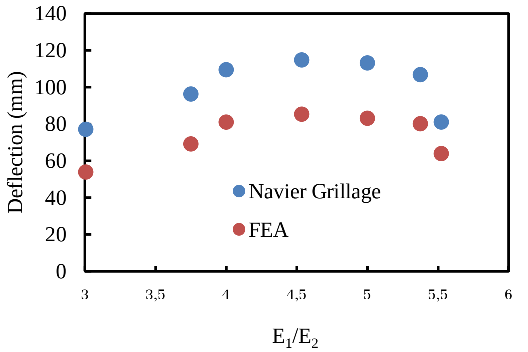

Figure 7. Deflection of e-glass composite grillages plotted against E_1/E_2

Validity of equation above can be increased by adding more materials and also by comparing the result with FEA.

CONCLUSION

To develop Navier Grillage theory especially for composites, the flaw must be found by performing a static analysis and the results were compared against FEA as a benchmark tool. The result shows that there are some limitations of the Navier Grillage for the composites material. The maximum difference between Navier Grillage code and FEA for carbon composites and E-glass composites is 12% (4.46 mm) and 26% (26.37 mm) respectively. Several parametric studies were conducted and can be concluded that the main cause of the difference between Navier Grillage and FEA is the material properties of the composite. The deflection analysis of elastic properties for several e-glass were performed and the result showed the trend of deflection graphs are similar between two methods. With the similarity in graph's trend, empirical equation for Navier Grillage for composite materials (e-glass) were derived by using the relationship between elastic properties of the composites.

REFERENCE

Biswal, M., Sahu, S. K. & Asha, A. V. (2017). Dynamic Stability of Woven Fiber Laminated Composite Shallow Shells in Hygrothermal Environment. *International Journal of Structural Stability and Dynamics*, Vol. 17(08): 1–26.

Blake, J. I. R., Shenoi, R. A., Das, P. K. & Yang, N. (2009). The Application of Reliability Methods in the Design of Stiffened FRP Composite Panels for Marine Vessels. *Ships and Offshore Structures*, Vol. 4(3): 287–297.

Blanchard, J., Mutlu, U., Sobey, A. J. & Blake, J. I. R. (2017). Empirically Improved Navier Grillage Theory for Assessment of Composite Top-hat Stiffened Structures. *Computers and Structures 6*.

Clarkson, J. (1965). *The Elastic Analysis of Flat Grillages With Particular Reference to Ship Structures*. Cambridge: Cambridge University Press.

Datoo, M. H. (1991). *Mechanics of Fibrous Composites*. New York: Elsevier Science Publishers Ltd.

Johnson A. F. & Sims, G. D. (1983). Simplified Design Procedures for Composite Plates under Flexural Loading. *Composite Structures 2*: 302–325.

Maneepan, K., Shenoi, R. A., Blake, J. I. R. & Jeong, H. K. (2007). Genetic Algorithms (GAs) based Optimisation of FRP Composite Plated Grillages in Ship Structures. *Transactions of the Royal Institution of Naval Architects Part A: International Journal of Maritime Engineering*, 149(3): 1–19.

Mutlu, U., Sobey, A. & Blake, J. (2016). Multiscale Reliability Approach to Analyse Composite Marine Structures. *International Conference on Structural Reliability of Ships, Offshore and Subsea Structures (SAROSS 2016)*.

Performance Composites. (2009). *Mechanical Properties of Carbon Fibre Composite Materials, Fibre/Epoxy resin (120°C Cure)*. Accessed on May 5th 2020. http://www.performance-composites.com/carbonfibre/mechanicalproperties_2.asp.

Raju, Prusty, B. G., Kelly, D. W., Lyons, D. & Peng, G. D. (2012). Top Hat Stiffeners: A Study on Keel Failures. *Ocean Engineering*, Vol. 37(13): 1180–1192.

Simonetta, P. (2019). *Boat Construction Composites & Eco-Compatibility*. Accessed on May 5th 2020 https://www.compositemood.com/Boat-Construc tion-Composites-Eco-Compatibility.

Sobey, A. J., Blake, J. I. R. & Shenoi, R. A. (2013). Implications of Failure Criteria Choices on the Rapid Concept Design of Composite Grillage Structures using Multiobjective Optimisation. *Structural and Multidisciplinary Optimization*, Vol. 47(5): 735–747.

Sobey, A., Blake, J. I. R. & Shenoi, R. A. (2008). Comparative Optimisation of First Principles Design Methods with Classification Society Rules for Boat Hull Structures. *Ship Technology Research*, Vol. 55: 502–516.

Sudheer, M., R, P. K., & Somayaji, S. (2015). Analytical and Numerical Validation of Epoxy/Glass Structural Composites for Elastic Models. *American Journal of Materials Science*, Vol. 5(3C): 162–168.

Upadhya, A. R. & Loughlan, J. (1981). The Effect of Mode Interaction in Orthotropic Fibre Reinforced Composite Plain Channel Section Columns. *Composite Structures 1*: 366–382.

Vedeler, G. (1945). *Grillage Beams in Ships and Similar Structures*. Norway: Grondhal & Son.

Yang, N., Das, P. K., Blake, J. I. R., Sobey, A. J. & Shenoi, R. A. (2013). The Application of Reliability Methods in the Design of Tophat Stiffened Composite Panels under In-plane Loading. *Marine Structures*, Vol. 32: 68–83.

Zhang, Y. & Matthews, F. L. (1983). Postbuckling Behaviour of Cylindrically Curved Panels of Generally Layered Composite Materials with Small Initial Imperfections of Geometry. *Composite Structures 2*: 428–441.